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ABSTRACT

This document is a list of approximately 450 mathematical concepts which are taught in grades K-8. The list is organized into eight major topics: (1) number systems, (2) numeration and notation, (3) sets, (4) geometry, (5) measurement, (6) number patterns and relationships, (7) other topics, and (8) summaries. The Content Authority List is used in conjunction with the Behavioral Objectives Authority List and the Vocabulary Authority List in the Pennsylvania Retrieval of Information for Mathematics Education System (PRIMES). This system of information storage and retrieval is used by local school districts in decision making with respect to curriculum, instruction, and evaluation. (SD)

PRIMES

Mathematics Content Authority List: K-8

Division of Research
Bureau of Information Systems
Bureau of Curriculum Services
Pennsylvania Department of Education

1974

2/3

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Robert B. Hayes, *Director*
Emanuel Berger, *Research Associate*
Judith L. Cope, *Research Assistant*
Dorothy Hoy Wasdovich, *Mathematics Education Adviser*

Bureau of Curriculum Services
Pauline M. Leet, *Director*

Division of Science and Technology
Irvin T. Edgar, *Chief*
Doris E. Creswell, *Mathematics Education Adviser*

Pennsylvania Department of Education
Box 911
Harrisburg, Pa. 17126

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INTRODUCTION

The Pennsylvania Retrieval of Information for Mathematics Education System has been established by the Department of Educattion to assist educators in curriculum development, implementation and evaluation in elementary school mathematics.

System

PRIMES is an information storage and retrieval system that uses computer technology. A comprehensive data base of analytical information about curriculum materials for school mathematics (grades K-8) has been stored in the computer and can be retrieved to meet the specifications of local school districts. Included in the data base is information about textbooks, achievement tests, curriculum practices, audio-visual materials and manipulative devices. Two models for individualization of instruction--flexible grouping and instructional kits--using the system's resources have been field tested.

The goals of PRIMES are:

1. To develop and maintain a data base of instruction and evaluation materials in elementary school mathematics.

2. To assist mathematics committees of local school districts in systematic curriculum development.

3. To effect change in classroom instruction by implementing the curriculum documents produced by the committees.

4. To develop teacher/administrator competencies in curriculum development, instruction and evaluation activities through system-related in-service workshops and graduate courses.

5. To develop and implement classroom models for individualization of instruction.

The data base is created by analyzing or classifying the instruction and evaluation materials using three analysis tools--a list of mathematics concepts and skills, *Mathematics Content Authority List*; a list of behavioral objectives, *Behavioral Objectives Authority List* and a list of mathematical terms, *Vocabulary Authority List*.

Services

The three authority lists--CAL, BOAL, VAL--are used by local school districts in accessing the data base for decision making in curriculum, instruction and evaluation. School districts receiving services are assigned a consultant who trains the committee members in using a set of curriculum procedures manuals. Services are available at regional centers or the Department of Education.

The system enables a school district to evaluate its current curriculum based on concepts and skills and performance objectives considered to have highest priority by its committee. The committee, again setting its own priorities, can construct a scope and sequence, select textbooks, manipulative devices, audio-visual and test materials, develop a teaching/reference guide and determine the relationship of standardized tests to its curriculum. PRIMES can also be applied to designating an individualized instructional program for open education geared to what each child knows and needs to learn.

Content Authority List

The *Mathematics Content Authority List* is a list of mathematics concepts that may be taught in the school mathematics curriculum, K-8. About 450 concepts have been identified and organized in outline form under eight major topics. A four-digit code has been assigned to each content for use with the computer.

The CAL has been used extensively since 1965 by trained analysts and this edition incorporates those suggestions which the editors approved.

The *Mathematics Content Authority List* (abridged) contains only concepts and the code numbers; the unabridged version includes definitions, explanations and examples.

The numbers in parentheses following selected concepts are references to other concepts that may be relevant.

CONTENTS

TOPIC VII: OTHER TOPICS

TOPIC VIII: SUMMARIES

TOPIC I:
Number Systems

Whole Numbers

BASIC CONCEPTS

Definition: set of whole numbers 0010

{0,1,2,3,4,5,6,...}

Developing cardinal number sense

Cardinal number expresses the *manyness* of a set; it tells *how many* elements are in a set.

Ex. N{a,b,c,d} = 4 Ex. N{ } = 0

A. Developing cardinal number zero 0020
(See 4070, 3060)

B. Developing cardinal numbers one through ten 0030
(See 3060, 0060)

C. Developing cardinal numbers beyond ten 0035
(See 3070, 0060)

Developing ordinal number sense 0040
(See 0075)

An ordinal number indicates the position of an item in a sequence of items in contrast to a cardinal number which tells how many items are in a set.

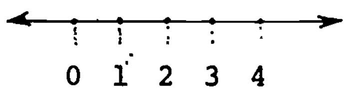
Ex. first, second, third; 4 o'clock (indicating the hour in a sequence of hours)

Whole Numbers

BASIC CONCEPTS

0050 Associating the idea of number with the number line (one-to-one correspondence)
(See 4010)

A one-to-one correspondence is said to exist between two sets A and B if every member of set A can be paired with a member of set B and every member of set B can be paired with a member of set A.

Ex. 

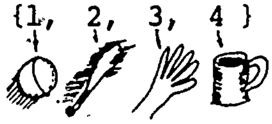
Each number on the number line corresponds to one point on the line or each number is paired with one point on the line.

0060 Counting to find cardinal number of set (one-to-one correspondence)
(See 0030, 0035, 4010)

Cardinal number - See page 1.

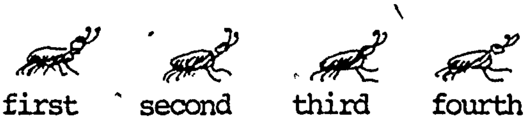
One-to-one correspondence - See 0050

Ex. {1, 2, 3, 4 }



0070 Ordinal counting

Ordinal number -- See 0040

Ex. 

0075 Sequence of numbers increasing by one
(See 0040, 7090)

Ex. 1,2,3,···
14,15,16,17,···
31,32,33,···

BASIC CONCEPTS

Skip counting 0080
(See 7000, 7055, 7090, 0380)

Ex. 2,4,6,8,···; 5,10,15,···; 5,8,11,14,···.

Other counting: backward, rote, etc. 0090

Ex. Backward: 9,8,7,6,···; 50,40,30,···;

Rote: 1,2,3,4,5; I caught a hare alive.
6,7,8,9,10; I let him go again.

Ordering numbers as greater than, less than, equal to or not equal to, and between; and objects as fewer than or more than 0100
(See 4010, 4030)

Ex. $7 > 2$; $5 < 9$; $3 + 1 = 4$; $2 \times 7 \neq 15$

7 apples are *more* than 5 apples.

A dog has *fewer* eyes than legs.

Trichotomy property

If a and b are whole numbers, then one and only one of the following statements is true: $a > b$, $a = b$, $b > a$. 0101

Ex. If $x \neq 3$, then $x > 3$ or $x < 3$.

OPERATIONS: ADDITION

Basic concepts

A. Addition, a binary operation 0110

A binary operation is an operation on two and only two elements in a set to produce a third element belonging to the set.

The binary operation of addition combines two numbers to obtain a unique result.

Ex. $2 + 3 = 5$ The number 2 and the number 3 are combined to obtain the number 5.

Whole Numbers

0120 *OPERATIONS: ADDITION*

B. Addition developed by using the union of disjoint sets or joining action
(See 4093)

Disjoint sets are sets which have no elements in common.

Ex.

{△, ○, □} and {▯, △} are disjoint sets.

The union of two disjoint sets forms a new set.

Ex.

{△, ○, □} ∪ {▯, △} = {△, ○, □, ▯, △}

Adding the numbers of two sets gives the number of the union of the sets.

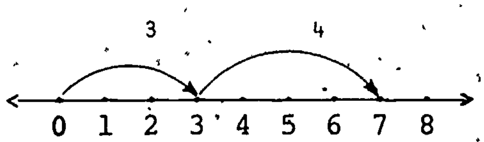
Ex.

N {△, ○, □} + N {▯, △} = N {△, ○, □, ▯, △}

3 + 2 = 5 (adding the cardinal numbers of the sets).

0130 C. Addition developed by using the number line

Ex.



3 + 4 = 7

0140 D. Closure, a property of addition

A binary operation with numbers such that the resulting number is always a member of the set being considered is said to be *closed* under that operation.

OPERATIONS: ADDITION

Since for every pair of numbers in a set of whole numbers, the unique sum is also in that set, closure is a property of addition of whole numbers.

Ex. A = {0,1,2,3,...}, the set of whole numbers

6 + 99 = 105
6 , 99 and 105 are all members of set A.

Ex. B = {1,3,5,7,...}, the set of odd whole numbers

3 + 5 = 8, but 8 is not in set B.
Closure is not a property of addition of odd numbers.

E. Commutativity, a property of addition 0150

Commutativity is another term for the commutative property.

A binary operation is said to possess commutativity if the result of combining two elements is independent of the order.

The commutative property is sometimes called the *order* property.

Ex. The operation of addition of whole numbers possesses commutativity since

2 + 3 = 3 + 2 = 5

$a + b = b + a$

F. Associativity, a property of addition 0160

Associativity is another name for the associative property.

An operation is said to possess associativity if the result of combining three elements by this operation is independent of the way in which the elements are grouped.

The associative property is sometimes called the *grouping* property.

Whole Numbers

OPERATIONS: ADDITION

Ex. The operation of addition of whole numbers is associative.

$$5 + 2 + 1 = (5 + 2) + 1 = 5 + (2 + 1)$$
$$7 \quad + 1 = 5 + \quad 3$$
$$8 = 8$$

Ex. $a + b + c = (a + b) + c = a + (b + c)$

0170 G. Zero, the identity element in addition

The identity element in addition is the number which when added to any number leaves that number unchanged.

0 is the identity element for addition.

Ex. $3 + 0 = 0 + 3 = 3$

Ex. $n + 0 = 0 + n = n$

0180 H. Role of one in addition

When 1 is added to a whole number the sum is the next greater whole number.

Ex. $23 + 1 = 24$ $\qquad$ $102 + 1 = 103$

Computation: two addends

0190 A. Elementary facts of addition

An elementary (basic) fact of addition has two whole number addends, each less than ten, and their sum.

Ex. $6 + 7 = 13$

OPERATIONS: ADDITION

B. Multi-digits used in addition without renaming (See 3010) 0200

Renaming *in addition* means considering ones as ones and tens, or tens as tens and hundreds, etc.

Ex.
$$\begin{array}{r} 213 \\ +\ \underline{142} \\ 355 \end{array}$$

No renaming is necessary. See 0210 for an example using renaming (in some texts called regrouping or carrying in addition).

C. Multi-digits used in addition with renaming (See 3010) 0210

Ex.
$$\begin{array}{rcl} 337 & = & 300 + 30 + 7 \\ +\ \underline{184} & = & \underline{100 + 80 + 4} \\ & & 400 + 110 + 11 \end{array}$$

$$\begin{aligned} &400 + 110 + 11 = \\ &400 + (100 + 10) + (10 + 1) = \\ &(400 + 100) + (10 + 10) + 1 = 521 \end{aligned}$$

The 11 ones are renamed as 1 ten and 1 one. The 11 tens are renamed as 1 hundred and 1 ten. The tens are combined and the hundreds are combined giving 521.

Computation: more than two addends

A. Single digits used in addition without renaming 0223

Ex.
$$\begin{array}{r} 3 \\ 2 \\ +\ \underline{4} \end{array} \qquad 5 + 6 + 4 = ?$$

3 + 2 + 4 does not use renaming since neither 5 + 4 nor 6 + 3 uses renaming. 5 + 6 + 4 does not use renaming since in neither 11 + 4 nor 5 + 10 do the ones need renamed.

OPERATIONS: ADDITION

0225 B. Single digits used in addition with renaming

When the addition fact used is greater than 9 + 9 renaming will be needed.

Ex.
```
  8
  9
  5
+ 3
```

Adding down 17 + 5 will use renaming even though addition may be done by considering the ending for the basic fact 7 + 5. That is, 12 will be renamed as 1 ten and 2 ones.

0227 C. Multi-digits used in addition without renaming
(See 3010)

Ex.
```
  213        313
  141        241
+ 234      + 631
  588       1185
```

The ones do not need to be renamed as tens and ones. The tens do not need to be renamed as hundreds and tens.

0229 D. Multi-digits used in addition with renaming
(See 3010)

Ex.
```
  427
  356
+ 485
 1268
```

18 ones must be considered as 1 ten and 8 ones, 16 tens as 1 hundred and 6 tens, etc.

0230 Historical algorithms.

Ex. The scratch method

```
66      367 + 298 = 665
555
367     The sum is recorded above the problem. Com-
298     putation is done from left to right, scratching
        out each partial sum as it is changed.
```

OPERATIONS: ADDITION

Use of addition tables 0231

Ex. Complete the addition table

+	0	1	2	3	4	5
0	0	1	2	3	4	
1	1	2		4		
2	2		4			
3	3	4				
4			6		8	
5	5	6				

Ex. Complete the addition table

+	1	3	5	7
1	2	4	6	
3	4			10
5				
7		10	12	

OPERATIONS: SUBTRACTION

Basic concepts

A. Subtraction, a binary operation 0240

Binary operation - See 0110

OPERATIONS: SUBTRACTION

0250 B. Subtraction developed in relation to subsets or separating action (See 4060)

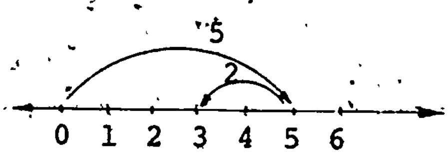
Ex. Pennies

Separate one penny from the set of five pennies.

$5 - 1 = 4$

0260 C. Subtraction developed from number line

Ex.



$5 - 2 = 3$

0270 D. Subtraction, the inverse of addition (relationship of addition and subtraction)

An inverse operation is one which undoes another operation; standing is the inverse of sitting and sitting is the inverse of standing.

Subtraction is the inverse of addition.

Ex. $8 + 3 = 11$ and $11 - 3 = 8$

Adding the number 3 to the number 8 gives the sum 11 ($8 + 3 = 11$).

Subtracting the number 3 from the sum 11 gives the missing addend 8 ($11 - 3 = 8$).

0280 E. Role of zero in subtraction

Zero is the right-hand identity element for subtraction.

Ex. $8 - 0 = 8$ $\quad n - 0 = n$

OPERATIONS: SUBTRACTION

Any number subtracted from itself is zero.

Ex. $8 - 8 = 0$ $n - n = 0$

F. Nonclosure, noncommutativity, nonassociativity of subtraction of whole numbers 0290

Closure - See 0140
Commutativity - See 0150
Associativity - See 0160

If $A = \{0,1,2,3,4,\ldots\}$ and the operation is subtraction, then closure is not a property of the operation.

Ex. $3 - 8 = {}^-5$ but ${}^-5$ is not a member of set A.

Commutativity is not a property of subtraction.

Ex. $7 - 3 \neq 3 - 7$ since $4 \neq {}^-4$

Associativity is not a property of subtraction.

Ex. $(9 - 2) - 1 \neq 9 - (2 - 1)$ since $7 - 1 \neq 9 - 1$

$6 \neq 8$

G. Role of one in subtraction

Subtracting 1 from a whole number gives the next lesser number. 0300

Ex. $7 - 1 = 6$ $36 - 1 = 35$

Computation

A. Elementary facts of subtraction 0310

An elementary (basic) fact of subtraction has *two* whole number addends each less than ten.

OPERATIONS: SUBTRACTION

Ex. $16 - 9 = 7$ $\quad 16 - \bigcirc = 7$

$16 - \square = 9$

$16 - 9 = \triangle$

The addends 9 and 7 are both less than 10.

0320 B. Multi-digits used in subtraction without renaming

Renaming *in subtraction* means to consider 1 ten as 10 ones or one hundred as 10 tens, etc.

Ex.
$$\begin{array}{r} 47 \\ -\underline{23} \\ 24 \end{array}$$
No renaming is necessary.

0330 C. Multi-digits used in subtraction with renaming (See 3010)

Ex.
$$\begin{array}{r} 52 \\ -\underline{25} \end{array}$$
5 tens and 2 ones may be renamed as 4 tens and 12 ones.

then
$$\begin{array}{r} 40 + 12 \\ -\underline{(20 + \ \ 5)} \\ 20 + \ \ 7 \text{ or } 27 \end{array}$$

0331 Historical algorithms

Ex. The scratch method

13
~~2~~~~1~~8
32~~7~~
189

$327 - 189 = 138$

The answer is written above the problem. Computation is done from left to right and numerals are scratched out as "borrowing" is done.

OPERATIONS: MULTIPLICATION

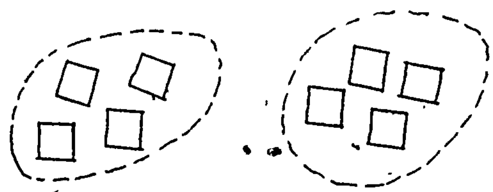
Basic concepts

A. Multiplication, a binary operation 0340

Binary operation - See 0110

B. Multiplication developed from union of two or more equivalent sets 0350

Ex.



Two sets of 4 are equivalent to one set of 8.
Two 4's are 8.
$2 \times 4 = 8$

C. Multiplication developed from arrays 0360

An array is an orderly arrangement of objects in rows and columns.

Ex. . . . .

. . . .

. . . .

Three 4's are 12 or $3 \times 4 = 12$

Ex. . . .

. . .

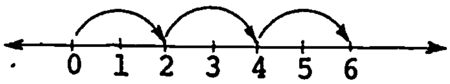
. . .

. . .

Four 3's are 12 or $4 \times 3 = 12$

0370 D. Multiplication developed from the number line

Ex.



Three 2's are 6.

$3 \times 2 = 6$

0380 E. Multiplication developed as repeated addition (See 0080)

Ex. $4 + 4 + 4 = 12$

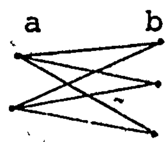
The sum of three 4's is 12.

$3 \times 4 = 12$

0390 F. Multiplication developed from Cartesian product sets

Cartesian product sets - See 4160

Ex.



How many dots in column *a*? in column *b*? Connect each dot in *a* with every dot in *b*. How many line segments did you draw? (6)

The first dot in column *a* is connected with 3 dots in column *b* by 3 different line segments. The second dot in column *a* is connected to the same 3 dots in column *b* by 3 different line segments. Together the dots are connected by 2×3 or 6 lines.

0400 G. Closure, a property of multiplication

Closure - See 0140

OPERATIONS: MULTIPLICATION

Ex. $12 \times 25 = 300$

12, 25 and 300 are all numbers in the set of whole numbers.

H. Commutativity, a property of multiplication 0410

Commutativity - See 0150

The product is independent of the order of the factors.

Ex. $2 \times 3 = 3 \times 2 = 6$

Ex. $a \times b = b \times a$

I. Associativity, a property of multiplication 0420

Associativity - See 0160

The product is independent of the way in which the factors are associated.

Ex. $3 \times 2 \times 1 = (3 \times 2) \times 1 = 3 \times (2 \times 1)$

Ex. $a \times b \times c = (a \times b) \times c = a \times (b \times c)$

J. Distributivity, a property of multiplication over addition or subtraction 0430

Distributivity is another name for the distributive property.

Multiplication is distributive over addition or subtraction.

Ex.
$$\begin{array}{rcl} 4 \times (3 + 2) & = & (4 \times 3) + (4 \times 2) \\ 4 \times 5 & = & 12 + 8 \\ 20 & = & 20 \end{array}$$

Ex.
$$\begin{array}{rcl} 4 \times (3 - 1) & = & (4 \times 3) - (4 \times 1) \\ 4 \times 2 & = & 12 - 4 \\ 8 & = & 8 \end{array}$$

Ex. $a \times (b + c) = ab + ac$

OPERATIONS: MULTIPLICATION

0440 K. One, the identity element in multiplication

Identity element - See 0170

One is the identity element for multiplication because multiplying a number by one leaves that number unchanged.

Ex. $5 \times 1 = 1 \times 5 = 5$

Ex. $1 \times 3 = 3 \times 1 = 3$

Ex. $n \times 1 = 1 \times n = n$

0450 L. Property of zero in multiplication

Any number times zero equals zero.

Ex. $5 \times 0 = 0 \times 5 = 0$

Ex. $n \times 0 = 0 \times n = 0$

Computation: two factors

0460 A. Elementary facts of multiplication

An elementary (basic) fact of multiplication has two whole number factors, each less than ten, and their product.

Ex. $5 \times 7 = 35$

0470 B. Multi-digits used in multiplication without renaming

Renaming in multiplication means considering ones as tens and ones, tens as hundreds and tens, etc.

Ex. $\begin{array}{r} 32 \\ \times\ 3 \\ \hline 96 \end{array}$ $\qquad$ $\begin{array}{r} 42 \\ \times\ 3 \\ \hline 126 \end{array}$

There is no need to consider ones as tens and ones nor tens as hundreds and tens.

OPERATIONS: MULTIPLICATION

C. Multi-digits used in multiplication with renaming 0480

$$\begin{array}{r} 45 \\ \times\ 7 \\ \hline 315 \end{array} \qquad \begin{array}{r} 40 + 5 \\ \times\ 7 \\ \hline 280 + 35 \end{array} =$$

$$280 + (30 + 5) =$$

$$(280 + 30) + 5 = 310 + 5 = 315$$

Note that 35 was considered as 3 tens + 5 ones. 3 tens were then added to 28 tens.

Computation: more than two factors

A. Multiplication with more than two factors, without renaming 0490

Only elementary facts are used.

Ex. $2 \times 3 \times 4$

Ex. $2 \times 3 \times 9$

Ex. $1 \times 2 \times 3 \times 4$

B. Multiplication with more than two factors, with renaming 0500

Ex. $6 \times 3 \times 9 = (6 \times 3) \times 9 = 6 \times (3 \times 9)$

$$18 \times 9 = 6 \times 27$$

$$162 = 162$$

When 8 is multiplied by 9 the 72 must be considered as 7 tens and 2 ones and 7 tens combined with 9 tens; or when 7 is multiplied by 6 the 42 must be considered as 4 tens and 2 ones and the 4 tens added to 12 tens.

Whole Numbers

OPERATIONS: MULTIPLICATION

Computation

0510 A. Multiples of ten as a factor

Multiples of 10 are numbers which have a factor of 10 such as 10, 20, 60, 120.

Ex. $\begin{array}{r} 12 \\ \times\ 10 \\ \hline 120 \end{array}$ $\begin{array}{r} 13 \\ \times\ 20 \\ \hline 260 \end{array}$ $\begin{array}{r} 25 \\ \times\ 60 \\ \hline 1500 \end{array}$

0515 B. A power of ten as a factor

Numbers such as 10^2, 10^3, 10^5, etc., are whole number powers of 10.

Ex. $100 \times 15 = 1{,}500$ $(100 = 10^2)$

Ex. $1000 \times 23 = 23{,}000$ $(1000 = 10^3)$

0520 C. A number expressed in exponential form as a factor (See 3120)

An exponent is a small numeral written above and to the right of a base numeral. When the exponent is a whole number it shows how many times the base is used as a factor.

Ex. $4^3 \times 5 = 4 \times 4 \times 4 \times 5$

Ex. $a^2 \times a^3 = a^{2+3} = a^5$

Ex. $6^2 \times 6^3 = 6^5$

Ex. $81 \times 3^3 = 3^4 \times 3^3 = 3^7$

Ex. $9^2 \times 3^3 = 3^2 \times 3^2 \times 3^3 = 3^7$

0521 Historical algorithms

Ex. "Peasant multiplication" is a method of multiplying by halving the multiplier (ignoring the remainder, if any) and doubling the multiplicand. Results are written in two columns, and any row with an even number on the left is crossed out. The total of the right hand column is the answer.

27

OPERATIONS: MULTIPLICATION

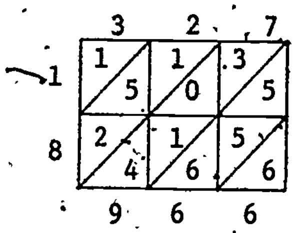
Multiply: 52 × 23

23	52
11	104
5	208
~~2~~	~~416~~
1	+832
	1196 (answer)

This method is based on the binary system.

Ex. The grid or grating method of multiplying

Multiply: 58 × 327



The product is 18,966.

Use of multiplication tables 0522

Ex. Complete the chart

×	1	2	3	4	5
1	1	2			
2					
3		6			
4					
5					

OPERATIONS: MULTIPLICATION

Ex. Use a multiplication table to find the answers to these problems:

$3 \times 8 =$

$2 \times 5 =$

OPERATIONS: DIVISION

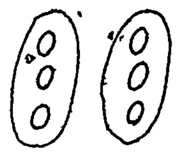
Basic concepts

0530 A. Division, a binary operation

Binary operation - See 0110

0540 B. Division developed from partitioning into equivalent sets (See 4060)

Ex.



How many sets of 3 objects each can be formed from 6 objects or if six objects are separated into two equivalent sets how large will each set be?

0550 C. Division developed as successive subtraction

Ex.
$$\begin{array}{r} 12 \\ -\underline{4} \\ 8 \\ -\underline{4} \\ 4 \\ -\underline{4} \\ 0 \end{array}$$

In the total operation 4 was subtracted 3 times with no remainder. There are three 4's in 12; $12 \div 4 = 3$.

29

OPERATIONS: DIVISION

D. Division developed from arrays 0555

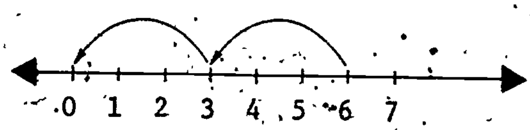
Arrays - See 0360

Ex. . . . .
. . . .

If an array of 8 members is arranged with 4 to a row, how many rows will there be? $8 \div 4 = 2$; or if an array of 8 members is arranged in 2 rows, how many items will be in each row? $8 \div 2 = 4$.

E. Division developed from the number line 0560

Ex.



How many 3's are there in 6?

$6 \div 3 = 2$

F. Division, the inverse of multiplication 0570

Inverse - See 0270

Ex. If $4 \times \square = 12$

then $12 \div 4 = \square$ (that same number)

G. Distributivity of division over addition or subtraction 0575

Ex. $(4 + 8) \div 2 = (4 \div 2) + (8 \div 2)$
$12 \div 2 = 6$ and $2 + 4 = 6$

Ex. $(8 - 2) \div 2 = (8 \div 2) - (2 \div 2)$
$6 \div 2 = 3$ and $4 - 1 = 3$

The distributivity of division over addition or subtraction holds only when the operation of addition or subtraction precedes the division.

0580 H. Role of one in division

Ex. $12 \div 1 = 12$; $n \div 1 = n$

One is the right-hand identity element for division.

Ex. $12 \div 12 = 1$; $n \div n = 1$

Any number except 0 divided by itself is 1.

0590 I. Zero not a divisor

What number could equal a number divided by zero?

$\frac{n}{0} = ?$ Could $\frac{6}{0} = 0$?

Since division is the inverse of multiplication then 0×0 would have to equal 6. This we know is not true. No number times 0 will equal 6 and no number times 0 will equal n. Therefore, division by zero is an undefined operation.

0600 J. Nonclosure, noncommutativity, nonassociativity of division

Closure - See 0140
Commutativity - See 0150
Associativity - See 0160

Ex. If A = $\{1, 2, 3, 4, \cdots\}$ and the operation is division, then:

closure is not a property of division since $3 \div 12 = \frac{1}{4}$, but $\frac{1}{4}$ is not a member of set A.

commutativity is not a property of division since $12 \div 3 \neq 3 \div 12$ as $4 \neq \frac{1}{4}$.

associativity is not a property of division since

$$(24 \div 8) \div 2 \neq 24 \div (8 \div 2)$$
$$3 \div 2 \neq 24 \div 4$$
$$\frac{3}{2} \neq 6$$

OPERATIONS: DIVISION

Computation

A. Elementary facts of division 0610

In elementary (basic) facts of division, both the known factor (divisor) and unknown factor (quotient) are whole numbers each less than ten.

Ex. $\begin{array}{r} 8 \\ 4\overline{)32} \end{array}$ or $32 \div 4 = 8$

B. Division: known factor (divisor) less than ten, product (dividend) not renamed; no remainder 0620

The dividend is not renamed if each digital value in the dividend is a multiple of the divisor.

Ex. $\begin{array}{r} 2341 \\ 2\overline{)4682} \end{array}$ $\begin{array}{r} 121 \\ 4\overline{)484} \end{array}$ $\begin{array}{r} 312 \\ 4\overline{)1248} \end{array}$

C. Division: known factor (divisor) less than ten, product (dividend) not renamed; remainder 0630

Ex. $\begin{array}{r} 11\text{ r }2 \\ 4\overline{)46} \end{array}$ $\begin{array}{r} 8\text{ r }1 \\ 4\overline{)33} \end{array}$

D. Division: known factor (divisor) less than ten; product (dividend) renamed; no remainder 0640

Ex. $\begin{array}{r} 32 \\ 8\overline{)256} \end{array}$

The dividend is renamed as 25 tens and 6 ones and then as 24 tens and 16 ones.

E. Division: known factor (divisor) less than ten, product (dividend) renamed; remainder 0650

Ex. $\begin{array}{r} 32\text{ r }1 \\ 8\overline{)257} \end{array}$

The dividend is renamed as 25 tens and 7 ones and then as 24 tens and 17 ones.

0660 F. Division by ten or greater numbers

```
Ex.      3         21             178 r 33
     10)30     26)546      296)52721
```

0665 G. Division by multiples of ten

```
Ex.      25          41            5           62 r 86
     10)250     40)1640     300)1500     120)7526
```

Note: If a text develops division as a series of subtractions the same type division exercises will be used though *renaming* as shown in 0640 will not be used.

The examples used in 0620 through 0665 might be solved as follows:

See 0620 -

```
Ex.    121
         1
        20
       100
    4)484
      400
       84
       80
        4
        4
```

See 0630 -

```
Ex.    11 r 2
        1
       10
    4)46
      40
       6
       4
       2
```

See 0650 -

```
Ex.    32 r 1
        2
       30
    8)257
      240
       17
       16
        1
```

See 0660 -

```
Ex.      21
          1
         20
    26)546
       520
        26
        26
```

See 0665 -

```
Ex.      25
          5
         20
    10)250
       200
        50
        50
```

OPERATIONS: DIVISION

H. Division by powers of ten 0667

Ex. $263 \div 100 = 2$ r 63

Ex. $4256 \div 1000 = 4$ r 256

Ex. $5000 \div 100 = 50$

I. Division with numbers expressed in exponential form 0670
(See 3120)

Exponents - See 0520

Ex. $10^5 \div 10^2 = 10^3$

Ex. $4^6 \div 4^2 = 4^4$

Use code 0670 only when exponents and bases are positive integers.

Historical algorithms 0671

Ex. The scratch or galley method

The modern algorithm is given along with the galley method. In the galley method, digits are crossed out and differences written above rather than below the minuend. The digits of a given numeral are not necessarily in the same row.

Divide 2631 by 37 Answer: 68 r 15

```
     6             6             68             68
37)2531       37)2531       37)2531        37)2531
   222           222           222            222
                 311           311            311
                               296            296
                                               15
```

```
                                                 1
                                                 12
             31            31                   315
37|2531|6    37|2531|6    37|2531|68       37|2531|68
  |222 |       |222 |       |2226|           |2226|
                            |  29|           |  29|
```

Whole Numbers

OPERATIONS

Combined operations (addition, subtraction, multiplication, division)

0680 A. Two sequential operations

Ex. $4 + 8 \div 2 = ?$

Parentheses should clarify such an example:

$4 + (8 \div 2) = 4 + 4 = 8$

$(4 + 8) \div 2 = 12 \div 2 = 6$

This code should not be used for examples such as:

a) $\begin{array}{r} 32 \\ \times \underline{18} \end{array}$ though both multiplication and addition are used.

b) $28\overline{)5656}$ though division, multiplication and subtraction are used.

0690 B. More than two sequential operations

See explanation 0680

Ex. $(4 \times 8) - 5 + (8 \div 2) =$

$32 - 5 + 4 = 31$

0700 Raising to powers and finding roots
(See 3120)

Ex. $4^3 = 4 \times 4 \times 4 = 64$

Ex. A square root of 25 is 5.

Note: Only whole numbers can be used in code 0700 since this topic is part of the major topic *Whole Numbers*. See page 1.

The square root of 25 is not considered as $25^{\frac{1}{2}}$. See 3120.

If the *notation* not operation is being developed, code 3120.

OPERATIONS

Several operations in the same lesson 0710

Note: If one or two operations predominate do not use this code. Code the operations.

More than two properties 0720

Note: If one or two properties predominate, do not use this code. Code the properties.

TOPIC I: Number Systems

NON-NEGATIVE RATIONAL NUMBERS

Fractional Numbers

BASIC CONCEPTS

1000 Definition: set of nonnegative rationals (fractional numbers)

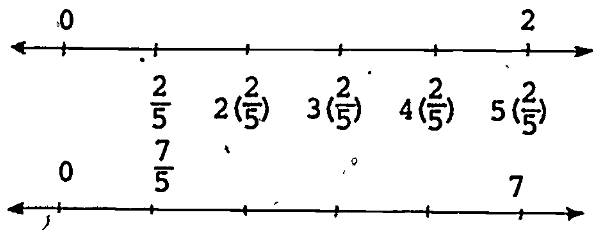
A nonnegative rational number is a number that can be expressed as the ratio of two whole numbers, provided that the second number (the divisor) is not zero.

Ex. $\frac{3}{4}$; $\frac{a}{b}$ when $b \neq 0$

Note: When nonnegative rational numbers are considered, the term *fractional number* will be used. When numerals are considered, the term *fraction* will be used in the content list.

1005 Developed in terms of basic operations

Ex.



Name the points shown on the number line in order.

$0, \frac{7}{5}, ? \times \frac{7}{5}, ? \times \frac{7}{5}, ? \times \frac{7}{5}, ? \times \frac{7}{5}$

$5 \times \frac{7}{5} = ?$

$\frac{7}{5}$ is $7 \div ?$

$7 \div 5$ is ?

37

BASIC CONCEPTS

Developed from arrays or subsets 1010

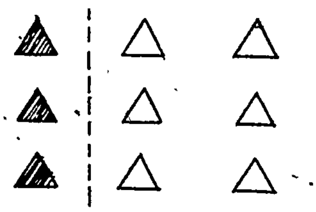
Ex. ● ○ ○ ○

Note the array of four circles. How many are shaded? When one out of four is shaded we say ¼ of the circles are shaded. Which numeral shows the total number of circles? Which shows the circle shaded?

Ex. $\{a,b\}$ is a subset of $\{a,b,c\}$.

How many elements are in the first set?
How many elements are in the second set?
We say that $\frac{2}{3}$ of the elements are in the subset.

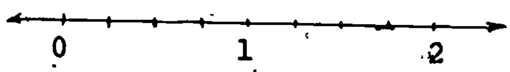
Ex. Look at the set of figures.



First see how many (equivalent) subsets are shown. Second, note the fraction name that tells what part of the whole set is shaded. (one-third)

Developed as distances on the number line 1020

Ex.



We can pair names for fractional numbers with points on the number line. What is the name of a point halfway between 0 and 1? Let us count by halves:

$$\frac{0}{2}, \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}$$

Fractional Numbers

BASIC CONCEPTS

Divide the space from 0 to 1 into fourths and count by fourths. When we write $\frac{3}{4}$ which numeral tells the number of equal parts into which the unit distance was divided? Which numeral tells the number of parts being considered?

1030 Developed from plane and solid regions

Ex. 

Into how many equal parts is the square shape divided? How many parts are shaded? What part of the whole is shaded? One of the two equal parts is shaded or one-half of the whole is shaded. Circular shapes, candy bars, cups and other familiar objects can be used to show fractional parts of a whole.

1035 Developed in other ways

1040 Whole numbers as related to set of nonnegative rationals (fractional numbers)
(See 3040)

The set of nonnegative rational numbers includes the set of whole numbers which may be written in fraction form $\frac{a}{b}$; a and b are whole numbers, a is a multiple of b and b is not zero.

Ex. $\frac{8}{2} = 4$ $\frac{12}{4} = 3$ $\frac{10}{2} = 5$ $\frac{6}{6} = 1$ $\frac{3}{1} = 3$

1060 Definition: equality
(See 3020, 3040)

Rational numbers which have the same value are equal.
$\frac{a}{b} = \frac{c}{d}$ if $ad = bc$ and bd is not zero.

Ex. The value of the fractional numbers $\frac{3}{8}$ and $\frac{6}{16}$ is the same since $3 \times 16 = 8 \times 6$.

BASIC CONCEPTS

Counting 1080

Ex. $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \ldots$

Ex. .2, .3, .4, .5, .6, ...

Ordering: greater than, less than, equal to, not equal to, between 1090

Equality - See 1060

Ex. $\frac{a}{b} > \frac{c}{d}$ if $ad > bc$ $\qquad \frac{2}{3} > \frac{7}{12}$

$\frac{a}{b} < \frac{c}{d}$ if $ad < bc$ $\qquad \frac{7}{8} < \frac{11}{12}$

$\frac{a}{b} = \frac{c}{d}$ if $ad = bc$ $\qquad \frac{2}{3} = \frac{4}{6}$

$\frac{1}{2} \square \frac{1}{3} \square \frac{1}{4}$ $\qquad \frac{7}{8} \square \frac{5}{6}$

Unit fraction; law of $\frac{1}{b}$
(See 1421)

A unit fraction is a fraction whose numerator is one and whose denominator is a positive integer. 1095

If a is a nonnegative integer, $a \times \frac{1}{b} = \frac{a}{b}$.

Ex. $4 \times \frac{1}{5} = \frac{4}{5}$

Trichotomy property 1098
(See 0101)

If a and b are nonnegative rational numbers, then one and only one of the following is true:

$$a < b, \ a = b, \ b > a$$

Fractional Numbers

BASIC CONCEPTS

1100 Density

Density is a term characterizing any ordered sequence of elements such that between any two elements of the sequence another element exists. Fractional numbers have density because between any two fractional numbers another fractional number exists.

Ex. Between $\frac{5}{10}$ and $\frac{6}{10}$ there exists another fractional number, such as $\frac{51}{100}$; between $\frac{5}{10}$ and $\frac{51}{100}$ there exists another fractional number, such as $\frac{101}{200}$; etc.

Ex. Between .8 and .9 there exists another decimal number, .81; between .8 and .81 there exists another decimal number, .807; etc.

OPERATIONS: ADDITION

Basic concepts

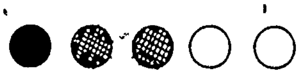
1110 A. Addition, a binary operation

Binary operation - See 0110

Ex. $\frac{1}{2} + \frac{3}{4} = ?$

Ex. $.45 + 2.13 = ?$

1120 B. Addition developed from union of disjoint sets

Ex. 

What part of the set of all the circle shapes is black? shaded? What part is marked in some way? Write a number sentence to show it.

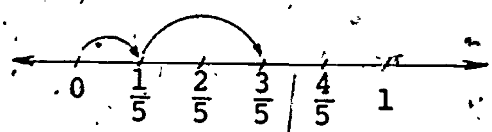
$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

For decimal fractions use 10 shapes.

OPERATIONS: ADDITION

C. Addition developed from the number line 1130

Ex.



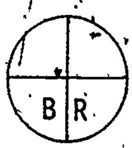
We can add fractional numbers with the same denominators as we added whole numbers.

$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

How shall we show the jumps? What is the denominator of the sum? How can you find the numerator of the sum?

D. Addition developed from plane or solid regions 1140

Ex.



What part of the circular shape is red? blue? What part is colored?

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$$

What does the 4 tell us? the 2? How could 2 be found from the numerators? Why must the 4 be used for all denominators? (It tells into how many equal parts the whole is separated.)

E. Closure, a property of addition 1150

Closure - See 0140.

Ex. $\frac{2}{3} + \frac{3}{8} = \frac{16}{24} + \frac{9}{24} = \frac{25}{24}$

Ex. $.3 + .5 = .8$

Fractional Numbers

OPERATIONS: ADDITION

1160 F. Commutativity, a property of addition

Commutativity - See 0150

Ex. $\frac{2}{3} + \frac{1}{4} = \frac{1}{4} + \frac{2}{3} = \frac{11}{12}$

Ex. $.3 + .5 = .5 + .3 = .8$

1170 G. Associativity, a property of addition

Associativity - See 0160

Ex. $\left(\frac{1}{4} + \frac{1}{3}\right) + \frac{1}{2} = \frac{1}{4} + \left(\frac{1}{3} + \frac{1}{2}\right)$

$\frac{7}{12} + \frac{1}{2} = \frac{1}{4} + \frac{5}{6}$

$\frac{7}{12} + \frac{6}{12} = \frac{3}{12} + \frac{10}{12} = \frac{13}{12}$

Ex. $.3 + (.25 + .4) = (.3 + .25) + .4$

$.3 + .65 = .55 + .4 = .95$

1180 H. Zero, the identity element in addition

Identity element - See 0170

Ex. $\frac{2}{3} + 0 = 0 + \frac{2}{3} = \frac{2}{3}$

Ex. $.4 + 0 = 0 + .4 = .4$

Computation

1190 A. Addition with common fraction notation, equal denominators (like fractions)

OPERATIONS: ADDITION

Ex. $\frac{3}{8} + \frac{1}{8} = \frac{4}{8}$ or $\frac{1}{2}$ or $\begin{array}{r} \frac{3}{8} \\ + \frac{1}{8} \\ \hline \frac{4}{8} \end{array}$ or $\frac{1}{2}$

Ex. $\begin{array}{r} 3 \\ + \frac{1}{4} \\ \hline 3\frac{1}{4} \end{array}$ Ex. $\begin{array}{r} 5\frac{1}{7} \\ + 2\frac{3}{7} \\ \hline 7\frac{4}{7} \end{array}$ Ex. $\begin{array}{r} 6 \\ 98\frac{1}{8} \\ 127\frac{3}{8} \\ + \frac{5}{8} \\ \hline 231\frac{9}{8} \end{array}$ or $232\frac{1}{8}$

B. Addition with common fraction notation, unequal denominators (unlike fractions) 1200
(See 7060)

Ex. $\frac{2}{3} + \frac{1}{2} = \square$ or $\begin{array}{r} \frac{2}{3} = \frac{4}{6} \\ + \frac{1}{2} = \frac{3}{6} \\ \hline \frac{7}{6} \end{array}$ or $1\frac{1}{6}$

$\frac{4}{6} + \frac{3}{6} = \frac{7}{6}$ or $1\frac{1}{6}$

Ex. $\begin{array}{r} 12\frac{7}{8} \\ + 6\frac{3}{4} \\ \hline 19\frac{5}{8} \end{array}$ Ex. $\begin{array}{r} \frac{3}{8} \\ \frac{2}{3} \\ + \frac{1}{6} \\ \hline 1\frac{5}{24} \end{array}$

OPERATIONS: ADDITION

1210 C. Addition with exact decimal fraction notation

Ex. .5 + .25 = .75 or

$$\begin{array}{r} .50 \\ +\ .25 \\ \hline .75 \end{array} \quad \text{(since .5 = .50)}$$

Ex.

$$\begin{array}{r} .121 \\ .342 \\ +\ .514 \\ \hline .977 \end{array}$$

Ex.

$$\begin{array}{r} 30.7 \\ 485.2 \\ +\ 1.96 \\ \hline 517.86 \end{array}$$

Do *not* use this code with addition involving money. Use 6040 and appropriate code under addition of whole numbers.

OPERATIONS: SUBTRACTION

Basic concepts

1220 A. Subtraction, a binary operation

Binary operation - See 0110

1230 B. Subtraction developed in relation to subsets

Ex.

What part of the set of all the circular shapes is 1 circular shape? 2? 3? 4? 5? What part of the shapes is shaded? Can we find the part not shaded by subtracting? How? $\frac{5}{5} - \frac{2}{5} = \frac{3}{5}$.

Check your answer by counting: $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}$ not shaded.

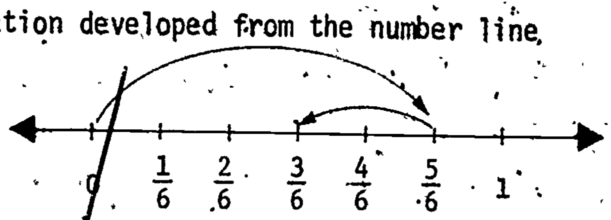
For decimal fractions use 10 shapes: $\frac{10}{10} - \frac{4}{10} = \frac{6}{10}$

or 1 - .4 = .6

OPERATIONS: SUBTRACTION

C. Subtraction developed from the number line 1240

Ex.



Count the parts shown on ~~the~~ number line. Subtract $\frac{2}{6}$ from $\frac{5}{6}$ as we did with whole numbers. $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$. How is the numerator found? Why is the denominator 6? (It tells into how many equal parts one unit of distance was divided in each case.) For decimal fractions divide the unit distance into 10 parts.

D. Subtraction developed from plane or solid regions 1250

Ex. See 1140

E. Subtraction, the inverse of addition 1260

Inverse - See 0270

Ex. If $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ then $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$ or $\frac{1}{2}$

Ex. If $\square + \frac{1}{4} = \frac{5}{4}$ then $\frac{5}{4} - \frac{1}{4} = \square$

Ex. If $\square + .31 = .56$ then $.56 - .31 = \square$

F. Role of zero in subtraction

Zero is the right identity element for subtraction. 1270

Ex. $\frac{3}{4} - 0 = \frac{3}{4}$ Ex. $\frac{a}{b} - 0 = \frac{a}{b}$

Ex. $.18 - 0 = .18$

Any number subtracted from itself is zero.

Ex. $\frac{2}{3} - \frac{2}{3} = 0$ Ex. $.26 - .26 = 0$

1280 G. Nonclosure, noncommutativity, nonassociativity in subtraction

Closure - See 0140
Commutativity - See 0150
Associativity - See 0160

Ex. Nonclosure:

$\frac{1}{2} - \frac{3}{4} = {}^{-}\frac{1}{4}$

$0.5 - 0.75 = {}^{-}0.25$

${}^{-}\frac{1}{4}$ and ${}^{-}0.25$ are not members of the set of nonnegative rational numbers.

Ex. Noncommutativity:

$\frac{1}{2} - \frac{3}{4} \neq \frac{3}{4} - \frac{1}{2}$

since ${}^{-}\frac{1}{4} \neq \frac{1}{4}$

$.25 - .13 \neq .13 - .25$

Ex. Nonassociativity:

$\left(\frac{1}{2} - \frac{1}{3}\right) - \frac{1}{6} \neq \frac{1}{2} - \left(\frac{1}{3} - \frac{1}{6}\right)$

$\frac{1}{6} - \frac{1}{6} \neq \frac{1}{2} - \frac{1}{6}$

$0 \neq \frac{2}{6}$

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OPERATIONS: SUBTRACTION

$$(.5 - .25) - .2 \neq .5 - (.25 - .2)$$

$$.25 - .2 \neq .5 - .05$$

$$.05 \neq .45$$

Computation

A. Subtraction with common fraction notation, equal denominators (like fractions) 1290

Ex. $\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$ or $\frac{2}{3}$

$$\begin{array}{r} \frac{5}{6} \\ -\frac{1}{6} \\ \hline \frac{4}{6} \text{ or } \frac{2}{3} \end{array}$$

Ex. $\begin{array}{r} 3 \\ -\frac{1}{2} \\ \hline 2\frac{1}{2} \end{array}$

Ex. $\begin{array}{r} 12\frac{3}{4} \\ -6\frac{1}{4} \\ \hline 6\frac{2}{4} \text{ or } 6\frac{1}{2} \end{array}$

B. Subtraction with common fraction notation, unequal denominators (unlike fractions) 1300
(See 7060)

Ex. $\frac{5}{6} - \frac{1}{2} = \frac{5}{6} - \frac{3}{6} = \frac{2}{6}$ or $\frac{1}{3}$ or $\begin{array}{rr} \frac{5}{6} = & \frac{5}{6} \\ -\frac{1}{2} = & \frac{3}{6} \\ \hline & \frac{2}{6} \text{ or } \frac{1}{3} \end{array}$

Ex. $\begin{array}{r} 6\frac{1}{2} \\ \frac{2}{3} \\ \hline 5\frac{5}{6} \end{array}$

Ex. $\begin{array}{r} 120\frac{11}{12} \\ 17\frac{1}{3} \\ \hline 103\frac{7}{12} \end{array}$

Fractional Numbers

OPERATIONS: SUBTRACTION

1310 C. Subtraction with decimal fraction notation

Ex. $\begin{array}{r} .75 \\ -\ .375 \\ \hline .375 \end{array}$ Ex. $\begin{array}{r} .584 \\ -\ .123 \\ \hline .461 \end{array}$

Do *not* use this code with subtraction involving money. Use 6040 and appropriate code under subtraction of whole numbers.

OPERATIONS: MULTIPLICATION

Basic concepts

1320 A. Multiplication, a binary operation

Binary operation - See 0110

Ex. $\frac{2}{3} \times \frac{6}{7} = \square$

Ex. $.3 \times .4 = \square$

1330 B. Multiplication developed from addition of two or more equal fractions

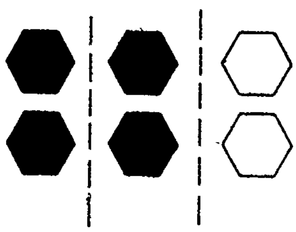
Ex. $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ Ex. $.3 + .3 = .6$

$2 \times \frac{1}{3} = \frac{2}{3}$ $2 \times .3 = .6$

1340 C. Multiplication developed from arrays or sets

Arrays - See 0360

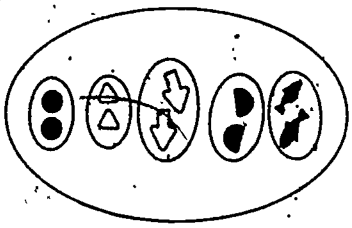
Ex.



$\frac{2}{3} \times 6 = 4$

OPERATIONS: MULTIPLICATION

Ex.

A = 

Six of the objects in set A are shaded.

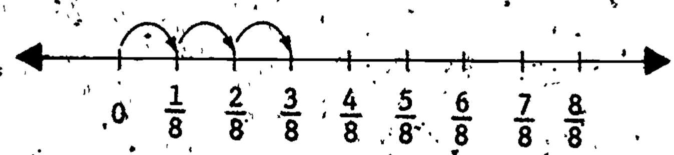
Three-fifths of the objects in set A are shaded.

$\frac{3}{5} \times 10 = 6$

Note: One of the factors will be a whole number.

D. Multiplication developed from the number line 1345

Ex.



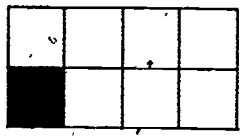
To find $3 \times \frac{1}{8}$ take 3 jumps of $\frac{1}{8}$ each on the number line $3 \times \frac{1}{8} = \frac{3}{8}$. Find $3 \times \frac{2}{8} = \frac{6}{8}$. How is the new numerator found? Find also $\frac{1}{2}$ of $\frac{5}{8}$. Where is the halfway point from 0 to $\frac{5}{8}$? $\left(\frac{5}{16}\right)$ Then $\frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$. How is the new numerator found? the new denominator?

Ex. For decimal fractions number the points as .1, .2, .3, etc. Two jumps of .3 each will bring the arrow to .6.

$2 \times .3 = .6$

E. Multiplication developed from plane and solid regions 1350

OPERATIONS: MULTIPLICATION

Ex. 

The shaded part is what part of the lower row? $\left(\frac{1}{4}\right)$; of the first column? $\frac{1}{2}$; of the whole figure? $\left(\frac{1}{8}\right)$. Is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ a true statement? How is the new denominator found? the new numerator?

Ex. In decimal fractions is $.5 \times .25 = .125$ a true statement? Is one square .125 of the whole?

1360 F. Closure, a property of multiplication

Closure - See 0140

1370 G. Commutativity, a property of multiplication

Commutativity - See 0150

Ex. $\frac{2}{3} \times \frac{4}{9} = \frac{4}{9} \times \frac{2}{3} = \frac{8}{27}$

Ex. $.5 \times .3 = .3 \times .5 = .15$

1380 H. Associativity, a property of multiplication

Associativity - See 0160

Ex. $\left(\frac{2}{3} \times \frac{1}{4}\right) \times \frac{1}{8} = \frac{2}{3} \times \left(\frac{1}{4} \times \frac{1}{8}\right)$

$$\frac{1}{6} \times \frac{1}{8} = \frac{2}{3} \times \frac{1}{32}$$

$$\frac{1}{48} = \frac{1}{48}$$

OPERATIONS: MULTIPLICATION

Ex. $.3 \times (.4 \times .5) = (.3 \times .4) \times .5$

$.3 \times .20 = .12 \times .5 = .060$

I. Distributivity, a property of multiplication over addition or subtraction 1390

Distributivity - See 0430

Ex. $\frac{2}{3} \times \left(\frac{1}{2} + \frac{1}{4}\right) = \left(\frac{2}{3} \times \frac{1}{2}\right) + \left(\frac{2}{3} \times \frac{1}{4}\right)$

$\frac{2}{3} \times \frac{3}{4} = \frac{1}{3} + \frac{1}{6}$

$\frac{1}{2} = \frac{1}{2}$

Ex. $\frac{2}{3} \times \left(\frac{1}{2} - \frac{1}{4}\right) = \left(\frac{2}{3} \times \frac{1}{2}\right) - \left(\frac{2}{3} \times \frac{1}{4}\right)$

$\frac{2}{3} \times \frac{1}{4} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

Ex. $.3 \times (.4 + .5) = (.3 \times .4) + (.3 \times .5)$

$.3 \times .9 = .12 + .15$

$.27 = .27$

J. One, the identity element in multiplication 1400

Identity - See 0170

Ex. $1 \times \frac{3}{8} = \frac{3}{8} \times 1 = \frac{3}{8}$

Ex. $1 \times .4 = .4 \times 1 = .4$

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1410 *OPERATIONS: MULTIPLICATION*

K. Role of zero in multiplication

Any number times zero is zero.

Ex. $\frac{a}{b} \times 0 = 0 \times \frac{a}{b}$ where $b \neq 0$

Ex. $\frac{2}{3} \times 0 = 0 \times \frac{2}{3} = 0$

Ex. $.3 \times 0 = 0 \times .3 = 0$

1420 L. Multiplicative inverse (reciprocal) for any fractional number greater than zero

If the product of two numbers is 1, then each number is the multiplicative inverse of the other.

The reciprocal of a number is its multiplicative inverse.

Ex. $3 \times \frac{1}{3} = 1$. 3 is the multiplicative inverse and the reciprocal of $\frac{1}{3}$; $\frac{1}{3}$ is the multiplicative inverse and the reciprocal of 3.

1421 M: Unit fraction property
(See 1095)

Ex. $\frac{1}{a} \times \frac{1}{b} = \frac{1}{a \times b}$

Ex. $\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

Computation

1430 A. Multiplication with common fraction notation

Ex. $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$ or $\frac{1}{6}$

OPERATIONS: MULTIPLICATION

Ex. $6 \times \frac{2}{3} = \frac{12}{3}$ or 4 $\quad \frac{2}{3}$ of $6 = \frac{12}{3}$ or 4

Ex. $7\frac{1}{2} \times \frac{5}{6} = \frac{15}{2} \times \frac{5}{6} = \frac{75}{12}$ or $6\frac{1}{4}$

Ex. $\frac{5}{6} \times (7 + \frac{1}{2}) = \frac{5}{6} \times 7 + \frac{5}{6} \times \frac{1}{2} = \frac{35}{6} + \frac{5}{12} = \frac{75}{12}$ or $6\frac{1}{4}$

B. Multiplication with decimal fraction notation 1440

Ex. $3 \times .18 = .54$

Ex. $.8 \times .3 = .24$

Ex. $.5 \times .25 = .125$ or $\begin{array}{r} .25 \\ \times\ .5 \\ \hline .125 \end{array}$

Do *not* use this code with multiplication involving money. Use 6040 and appropriate code under multiplication of whole numbers.

C. Multiplication by powers or multiples of ten 1441

Ex. $10^2 \times \frac{7}{100} = 10^2 \times .07$

10^2 is written as a power of ten.

Ex. $30 \times \frac{7}{10} = 21$

30 is a multiple of 10.

Ex. $300 \times \frac{7}{100} = 21$

300 is a multiple of 10.

Fractional Numbers

OPERATIONS: DIVISION

Basic concepts

1450 A. Division, a binary operation

Binary - See 0110

1460 B. Division developed from successive subtraction of two or more equal fractional numbers

Ex. $\frac{3}{4} \div \frac{1}{4} = \square$

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$$

$$\frac{2}{4} - \frac{1}{4} = \frac{1}{4}$$

$$\frac{1}{4} - \frac{1}{4} = 0$$

Three $\frac{1}{4}$ths can be subtracted from $\frac{3}{4}$ with no remainder,

$\frac{3}{4} \div \frac{1}{4} = 3.$

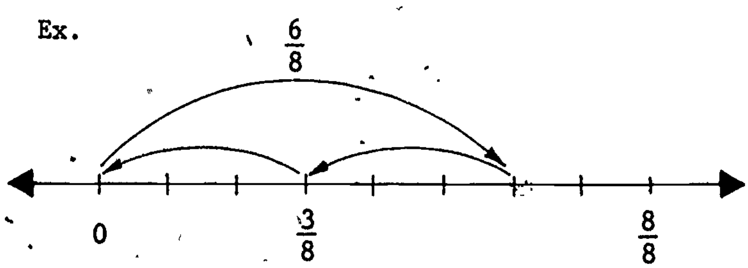
Ex.
$$\begin{array}{r} .2\overline{).8} \\ -\underline{.2} \\ .6 \\ -\underline{.2} \\ .4 \\ -\underline{.2} \\ .2 \\ -\underline{.2} \\ 0 \end{array}$$

$$\begin{array}{r} 4 \\ .2\overline{).8} \end{array}$$

$.8 \div .2 = 4$

1470 C. Division developed from the number line

Ex.



OPERATIONS: DIVISION

$\frac{6}{8} \div \frac{3}{8} = \boxed{}$ How many $\frac{3}{8}$ are there in $\frac{6}{8}$?

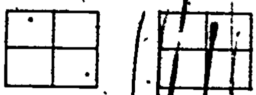
What is $6 \div 3$? $8 \div 8$?

Does $\frac{2}{1}$ name the same number as 2?

Ex. $\frac{5}{8} \div \frac{2}{16} = \frac{10}{16} \div \frac{2}{16} = \frac{5}{1}$ or 5.

This may lead to $\frac{5}{8} \times \frac{16}{2} = \frac{5}{1}$ so that the problem $\frac{5}{8} \div \frac{2}{16}$ may be solved by finding $\frac{5}{8} \times \frac{16}{2}$.

D. Division developed from plane and solid regions 1480

Ex. 

How many fourths are there in 2 whole figures?

$$2 \div \frac{1}{4} = 8$$

E. Division, the inverse of multiplication with fractional numbers. 1490

Ex. $\frac{3}{8} \div \frac{1}{2} = \frac{6}{8}$

$\frac{3}{8} \times ? = \frac{6}{8}$

How is 2 related to $\frac{1}{2}$? Write a multiplication sentence for $5 \div \frac{1}{3}$. Does the multiplication sentence solve the division sentence?

OPERATIONS: DIVISION

1500 F. Closure, a property of division

Closure - See 0140

1510 G. Role of one in division

One is the right hand identity element for division

Ex. $\frac{a}{b} \div 1 = \frac{a}{b}$

Ex. $.8 \div 1 = .8$

Any number divided by itself is one. (Note exception, see 1520.)

Ex. $\frac{a}{b} \div \frac{a}{b} = 1$

Ex. $.6 \div .6 = 1$

1520 H. Zero not a divisor
(See 0590)

Could $\frac{2}{3}$ be divided by 0? No.

There is no number times 0 which will equal $\frac{2}{3}$.

1530 I. Noncommutativity, nonassociativity of division

Commutativity - See 0150
Associativity - See 0160

Ex. Noncommutativity $\frac{2}{9} \div \frac{1}{3} \neq \frac{1}{3} \div \frac{2}{9}$

$$\frac{2}{9} \times \frac{3}{1} \neq \frac{1}{3} \times \frac{9}{2}$$

$$\frac{2}{3} \neq \frac{3}{2}$$

OPERATIONS: DIVISION

Ex. Nonassociativity $\left(\frac{2}{9} \div \frac{1}{3}\right) \div \frac{1}{2} \neq \frac{2}{9} \div \left(\frac{1}{3} \div \frac{1}{2}\right)$

$$\left(\frac{2}{9} \times \frac{3}{1}\right) \div \frac{1}{2} \neq \frac{2}{9} \div \left(\frac{1}{3} \times \frac{2}{1}\right)$$

$$\frac{2}{3} \div \frac{1}{2} \neq \frac{2}{9} \div \frac{2}{3}$$

$$\frac{2}{3} \times \frac{2}{1} \neq \frac{2}{9} \times \frac{3}{2}$$

$$\frac{4}{3} \neq \frac{1}{3}$$

Computation

A. Division with common fraction notation 1540

Ex. $\frac{2}{9} \div \frac{1}{3} = \square$ if $\square \times \frac{1}{3} = \frac{2}{9}$

$$\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$

therefore $\frac{2}{9} \div \frac{1}{3} = \frac{2}{3}$

$\frac{2}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{1} = \frac{2}{3}$ and therefore $\frac{2}{9} \div \frac{1}{3} = \frac{2}{9} \times \frac{3}{1} = \frac{2}{3}$

B. Division with decimal fraction notation 1550

Ex. $.5 \div .25 = \square$ if $\square \times .25 = .5$

OPERATIONS: DIVISION

$$2 \times .25 = .5$$

therefore $.5 \div .25 = 2$

or $.25\overline{).50}$ = $2.$

Check: $2 \times .25 = .50$

Ex. $.35\overline{)10.535}$ = 30.1

Check: $.35 \times 30.1 = 10.535$

Do *not* use this code with division involving money. Use 6040 and appropriate code used under division of whole numbers.

1555 C. Division by powers or multiples of ten

Ex. $.563 \div 10^2 = .563 \div 100 = .00563$

Ex. $\frac{1}{2} \div 10^3 = \frac{1}{2} \div 1000$

$$\frac{1}{2} \times \frac{1}{1000} = \frac{1}{2000}$$

OPERATIONS

1560 Sequential operations

This coding should be used when two or more sequential operations are indicated in operational format.

Ex. $\frac{3}{4} \times \left(\frac{2}{3} \div \frac{1}{6}\right) = \frac{3}{4} \times \left(\frac{2}{3} \times \frac{6}{1}\right)$

$$\frac{3}{4} \times 4 = 3$$

Ex. $.2 \times (.12 \div .3) = .2 \times .4 = .08$

OPERATIONS

Several operations in the same lesson 1610

Note: If one or two operations predominate, do not use this code. Code the operations.

More than two properties 1620

Note: If one or two properties predominate, do not use this code. Code the properties.

TOPIC I:

Number Systems

Integers

BASIC CONCEPTS

2000 Definition: set of integers

All of the numbers 0, ± 1, ± 2, ± 3, ... form the set of integers.

The set that contains every natural number, its additive inverse and zero is the set of integers.

2010 Developed from the number line

Ex.

$^{-}3$ $^{-}2$ $^{-}1$ 0 1 2 3

2020 Developed from physical world situations

Ex. A thermometer uses a number line in vertical position. Show 10 degrees below zero on the number line. This is often written as $^{-}10°$. What does $^{-}20°$ mean? $^{+}40°$?

Games like shuffleboard and monopoly use negative integers to indicate the player "owes" a score or play money. A disk may land on 10 OFF ($^{-}10$) or a player may be in debt \$20 ($^{-}20$).

2030 Ordering: greater than; less than; equal to or not equal to; between

When the number line is in a horizontal position each numeral to the right of another numeral represents a greater number. Is 5 > 4? (Yes). Is $^{-}5$ > $^{-}4$? (No). Does $^{-}4$ lie to the right of $^{-}5$? (Yes). Is the number represented by $^{-}5$ < the number represented by $^{-}4$? (Yes). What integer lies between $^{-}6$ and $^{-}8$?

BASIC CONCEPTS

Directed numbers: absolute value 2040

Directed numbers are also called positive and negative numbers.

Ex. On a horizontal number line positive numbers are usually indicated to the right of zero. Then negative numbers are indicated to the left of zero. When the positive direction is determined on a vertical or slanted number line the negative direction will be opposite to it. Zero is the point from which other number points are established and is considered to be neither positive nor negative.

The absolute value of a positive number is that number. Notation $|{}^{+}3| = {}^{+}3$.

The absolute value of a negative number is its additive inverse. Notation $|{}^{-}3| = {}^{+}3$.

The absolute value of both ${}^{-}2$ and ${}^{+}2$ is ${}^{+}2$.

On a number line the absolute value of a number is shown by its distance from zero without regard to the direction.

OPERATIONS: ADDITION

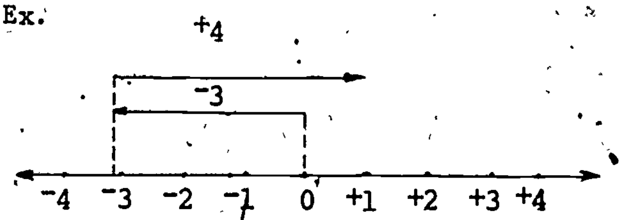
Basic concepts

A. Addition, a binary operation 2050

Binary - See 0110

B. Addition developed from number line 2053

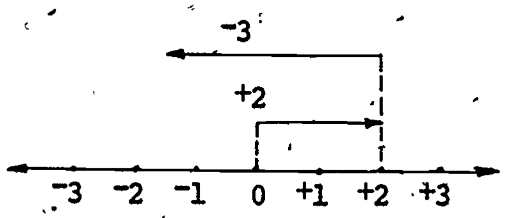
Ex.



$${}^{-}3 + ({}^{+}4) = {}^{+}1$$

OPERATIONS: ADDITION

Ex.



$^{+}2 + (^{-}3) = {}^{-}1$

2055 C. Addition developed from physical world situations

Ex. If the thermometer shows $^{-}10°$ and then rises 15°, what is the temperature? $^{-}10 + (^{+}15) = {}^{+}5$. The temperature is $^{+}5°$.

Ex. If you owe \$5 and pay \$3 what is your financial standing? $^{-}5 + (^{+}3) = {}^{-}2$ (still owe \$2).

Ex. If a submarine is 50 feet below the surface and then goes down 20 feet, what is its position? $^{-}50 + (^{-}20) = {}^{-}70$ (70 feet below sea level).

2060 D. Closure, a property of addition

Closure - See 0140

2070 E. Commutativity, a property of addition

Commutativity - See 0150

Ex. $^{-}3 + (^{+}4) = {}^{+}4 + (^{-}3)$

2080 F. Associativity, a property of addition

Associativity - See 0160

Ex. $(^{-}3 + {}^{+}4) + (^{-}2) = {}^{-}3 + (^{+}4 + {}^{-}2)$

$^{+}1 + (^{-}2) = -3 + (+2)$

$^{-}1 = {}^{-}1$

OPERATIONS: ADDITION

G. Zero, the identity element in addition 2090

Identity - See 0170

Ex. $^{-}6 + 0 = 0 + (^{-}6) = {}^{-}6$

$^{+}6 + 0 = 0 + (^{+}6) = {}^{+}6$

H. Additive inverse 2100

The additive inverse of any number is a second number which if added to the first number gives the sum of zero. For each integer a the additive inverse is ^{-}a.

Ex. $^{+}3 + (^{-}3) = 0$

$(^{-}4) + (^{+}4) = 0$

Computation 2110

See 205[illegible]

Ex. $^{+}3 + (^{+}8) = {}^{+}11$

$^{+}3 + (^{-}8) = {}^{-}5$

$^{-}3 + (^{+}8) = {}^{+}5$

$^{-}3 + (^{-}8) = {}^{-}11$

Do you see any pattern for these sums?

OPERATIONS: SUBTRACTION

Basic concepts

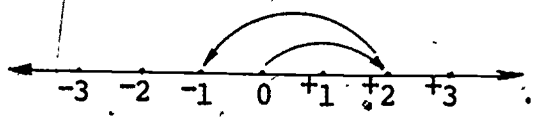
A. Subtraction, a binary operation 2120

Binary - See 0110

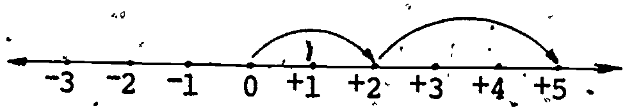
B. Subtraction developed from number line 2123

OPERATIONS: SUBTRACTION

Ex. $^{+}2 - (^{+}3) = ^{-}1$



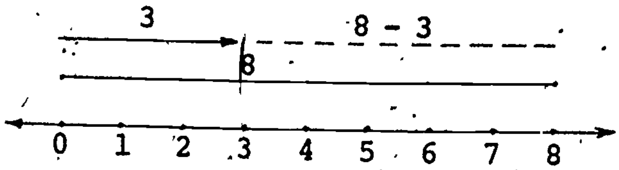
Ex. $^{+}2 - (^{-}3) = ^{+}5$



Subtracting a positive 3 on the number line was shown by movement to the left so subtracting a negative 3 must be shown by movement to the right of the positive 2.

A second explanation is possible when subtraction is considered as finding the difference between two numbers, or on the number line as finding distance between two number points.

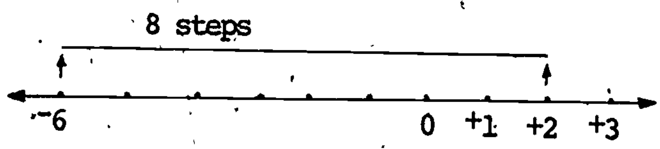
Ex. 8 - 3 = 5



To find the difference between 3 and 8 on the number line we may ask, "How far is it from 3 to 8?" or think 8 - 3 = 5. We may think it is 5 steps in the positive direction from 3 to 8.

OPERATIONS: SUBTRACTION

Ex. $^{+}2 - (^{-}6) = {}^{+}8$



To find the distance from $^{-}6$, the known addend, to $^{+}2$, the sum, move 8 steps from $^{-}6$ to $^{+}2$ in the positive direction showing the difference to be $^{+}8$.

Note: The distance and the direction are found in going from the known addend (subtrahend) to the sum (minuend).

C. Subtraction developed from physical world situations 2125

Ex. If you had \$7 and bought something for \$10 you must subtract 10 from 7 to find your financial standing. Your standing is $^{+}7 - (^{+}10) = {}^{-}3$ or you will be \$3 in debt.

Ex. If you owed \$5 and subtracted \$2 of that debt, what is your financial standing? Taking away a debt is equivalent to adding the money so you may think $^{-}5 - (^{-}2) = {}^{-}5 + (^{+}2) = {}^{-}3$. You still owe \$3.

D. Subtraction, the inverse of addition 2130

Inverse - See 0270

Ex. $^{+}5 - (^{-}2) = {}^{+}7$ because $^{+}7 + (^{-}2) = {}^{+}5$

Ex. $^{-}5 - (^{+}2) = {}^{-}7$ because $^{-}7 + (^{+}2) = {}^{-}5$

E. Role of zero in subtraction 2140

Zero is the right identity element for subtraction.

Ex. $^{-}3 - 0 = {}^{-}3$

$^{+}4 - 0 = {}^{+}4$

OPERATIONS: SUBTRACTION

Any number subtracted from itself is zero.

Ex. $^{-}3 - (^{-}3) = 0$

$n - n = 0$

Any number subtracted from zero results in the additive inverse of that number.

Ex. $0 - (^{+}3) = {}^{-}3$

$0 - (^{-}3) = {}^{+}3$

2150 F. Closure, a property of subtraction

Closure - See 0140

2160 G. Noncommutativity, nonassociativity of subtraction

Commutativity - See 0150
Associativity - See 0160

Ex. Noncommutativity

$^{-}2 - (^{-}5) \neq {}^{-}5 - (^{-}2)$

$^{+}3 \neq {}^{-}3$

Ex. Nonassociativity

$(^{+}3 - {}^{-}2) - {}^{+}5 \neq {}^{+}3 - (^{-}2 - {}^{+}5)$

$^{+}5 - (^{+}5) \neq {}^{+}3 - (^{-}7)$

$0 \neq {}^{+}10$

2170 Computation

Ex. $^{-}3 - (^{+}4) = {}^{-}7$

Think "How far is it from $^{+}4$ to $^{-}3$ and in what direction?"; or think $^{-}3 - (^{+}4)$ is equivalent to $^{-}3 + (^{-}4) = {}^{-}7$.

OPERATIONS: SUBTRACTION

Ex. $^-3 - (^+4) = {}^-7$

$^-3 - (^-4) = {}^+1$

$^+3 - (^+4) = {}^-1$

$^+3 - (^-4) = {}^+7$

OPERATIONS: MULTIPLICATION

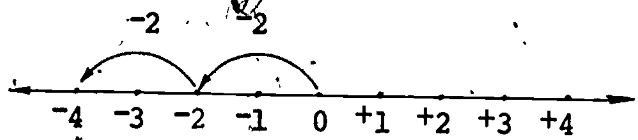
Basic concepts

A. Multiplication, a binary operation 2180

Binary - See 0110

B. Multiplication developed from number line 2183

Ex.

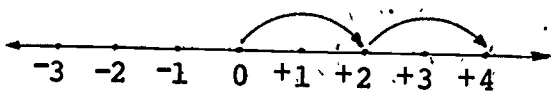


$^+2 \times {}^+2 = {}^+4$ (already known)

$^+2 \times (^-2) = {}^-4$ (drawn on the number line)

$^-2 \times (^+2) = {}^-4$ (see 2200, multiplication is commutative)

Ex.



$^-2 \times (^-2)$ must be drawn in a direction opposite to $^+2 \times (^-2)$ and equals $^+4$.

C. Multiplication developed from physical world situations 2185

Ex. If you have 2 × \$4 what is your financial standing? $^+2 \times (^+4) = {}^+8$. You have \$8.

68

OPERATIONS: MULTIPLICATION

Ex. If you spend \$4 each week and receive no additional funds what is your financial standing after two weeks have passed? $^{+}2 \times (^{-}4) = {}^{-}8$. You will have \$8 less than you have now.

Ex. If you spend \$4 each week what was your financial standing two weeks ago? $^{-}2 \times (^{-}4) = {}^{+}8$. You had \$8 more two weeks ago than you have now if you received no additional funds.

2190 D. Closure, a property of multiplication

Closure - See 0140

2200 E. Commutativity, a property of multiplication

Commutativity - See 0150

Ex. $(^{-}3) \times (^{+}8) = (^{+}8) \times (^{-}3)$

2210 F. Associativity, a property of multiplication

Associativity - See 0160

Ex. $(^{-}3 \times {}^{+}2) \times {}^{+}5 = {}^{-}3 \times ({}^{+}2 \times {}^{+}5)$

$$^{-}6 \times {}^{+}5 = {}^{-}3 \times {}^{+}10$$

$$^{-}30 = {}^{-}30$$

2220 G. One, the identity element in multiplication

Identity element - See 0170

Ex. $^{+}1 \times {}^{-}5 = {}^{-}5 \times {}^{+}1 = {}^{-}5$

$^{+}1 \times {}^{+}5 = {}^{+}5 \times {}^{+}1 = {}^{+}5$

Multiplying by $^{+}1$ leaves the number ($^{+}5$ or $^{-}5$) unchanged.

OPERATIONS: MULTIPLICATION

H. Distributivity, a property of multiplication over addition or subtraction 2225

Ex. Rewrite the example to make the work easier, and then find the correct replacement for *N*.

$(48 \times {}^{-}25) + ({}^{-}25 \times 52) = N$

${}^{-}25 \times (48 + 52) = {}^{-}2500$

Computation 2230

Ex. $\begin{array}{r} {}^{+}2 \\ \times\ {}^{+}5 \\ \hline {}^{+}10 \end{array} \quad \begin{array}{r} {}^{-}2 \\ \times\ {}^{+}5 \\ \hline {}^{-}10 \end{array} \quad \begin{array}{r} {}^{-}2 \\ \times\ {}^{-}5 \\ \hline {}^{+}10 \end{array} \quad \begin{array}{r} {}^{+}2 \\ \times\ {}^{-}5 \\ \hline {}^{-}10 \end{array}$

What pattern do you see for these products?

OPERATIONS: DIVISION

~~Basic~~ concepts

A. Division, a binary operation 2240

Binary - See 0110

B. Division developed from number line 2243

Ex. $\longleftrightarrow$ ${}^{-}4 \quad {}^{-}3 \quad {}^{-}2 \quad {}^{-}1 \quad 0$

If the space from 0 to ${}^{-}4$ is divided by 2, what point is determined? $\frac{{}^{-}4}{{}^{+}2} = {}^{-}2$. Then $\frac{{}^{-}4}{{}^{-}2}$ must determine point ${}^{+}2$. Check both examples by multiplying.

C. Division developed from physical world situations. 2245

OPERATIONS: DIVISION

Ex. The temperature fell from $^-4$ to $^-12$ degrees in 2 hours. What was the average change per hour?

$$\frac{^-8}{^+2} = {}^-4$$

Ex. The temperature rose from $^-8$ to $^-2$ in 3 hours. The temperature rose 6 degrees in 3 hours or

$$\frac{6}{3} \text{ or } 2 \text{ degrees per hour.}$$

Ex. The temperature is zero (0°) now. Two hours ago ($^-2$) it was 8 degrees below zero ($^-8$). What was the average change per hour?

$$\frac{^-8}{^-2} = {}^+4$$

2250 D. Division, the inverse of multiplication

Ex. $\frac{^-10}{^-2} = {}^+5$ because $^+5 \times {}^-2 = {}^-10$

$\frac{^-10}{^+2} = {}^-5$ because $^-5 \times {}^+2 = {}^-10$

$\frac{^+10}{^-2} = {}^-5$ because $^-2 \times {}^-5 = {}^+10$

2255 E. Role of one in division

$^+1$ is the right identity element for division.

Ex. $^-3 \div {}^+1 = {}^-3$

Any number divided by itself is $^+1$. (Note exception, see 1520.)

Ex. $\frac{^-3}{^-3} = {}^+1$

Ex. $\frac{n}{n} = {}^+1 \qquad n \neq 0$

OPERATIONS: DIVISION

F. Nonclosure, noncommutativity, nonassociativity of division 2260

Closure - See 0140
Commutativity - See 0150
Associativity - See 0160

Ex. Nonclosure

$$^{-}3 \div {}^{+}2 = \frac{^{-}3}{^{+}2} \text{ (not an integer)}$$

Ex. Noncommutativity

$$^{-}8 \div {}^{+}2 \neq {}^{+}2 \div {}^{-}8$$

$$^{-}4 \neq {}^{-}\frac{1}{4}$$

Ex. Nonassociativity

$$(^{-}8 \div {}^{+}4) \div {}^{+}2 \neq {}^{-}8 \div ({}^{+}4 \div {}^{+}2)$$

$$^{-}2 \div {}^{+}2 \neq {}^{-}8 \div {}^{+}2$$

$$^{-}1 \neq {}^{-}4$$

Computation 2270

Ex. $\frac{^{-}12}{^{+}4} = {}^{-}3$ Check $^{-}3 \times {}^{+}4 = {}^{-}12$

$\frac{^{-}12}{^{-}4} = {}^{+}3$ Check $^{+}3 \times {}^{-}4 = {}^{-}12$

$\frac{^{+}12}{^{-}4} = {}^{-}3$ Check $^{-}3 \times {}^{-}4 = {}^{+}12$

$\frac{^{+}12}{^{+}4} = {}^{+}3$ Check $^{+}3 \times {}^{+}4 = {}^{+}12$

Do you see any pattern for these factors?

Integers

OPERATIONS

2310 Several operations in the same lesson

Note: If one or two operations predominate, do not use this code. Code the operations.

2320 More than two properties

Note: If one or two properties predominate, do not use this code. Code the properties.

TOPIC I:
Number Systems
Rational Numbers

BASIC CONCEPTS

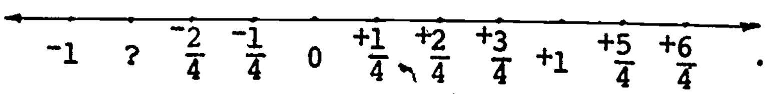
Definition of a set of rationals 2520

Rational numbers are numbers which can be expressed as a ratio of two integers, $\frac{a}{b}$, where b cannot be 0. When either a or b is negative the number is a negative rational.

Ex. $-\frac{3}{4}$; $\frac{-3}{4}$ Both fractions may be written as $\frac{^-3}{4}$.

Developed from number line 2530

Ex.



Absolute value 2535
(See 2040)

Ex. $\left|\frac{^-2}{3}\right| = \frac{2}{3}$

Ex. $|^+1.43| = 1.43$

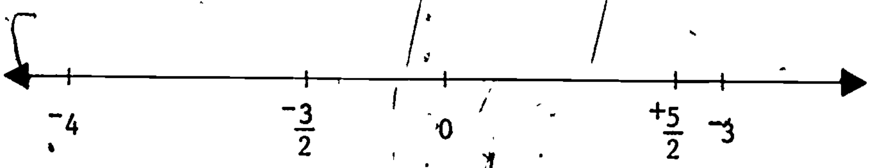
Ordering 2540
(See 2030)

Ex. Graph the following rational numbers on a number line:

Rational Numbers

BASIC CONCEPTS

$^{+}3,\ ^{-}\frac{3}{2},\ ^{+}\frac{5}{2},\ 0,\ ^{-}4$



Ex. Which fraction names the greater number, $^{-}\frac{4}{7}$ or $^{-}\frac{3}{5}$?

$^{-}\frac{4}{7} = ^{-}\frac{20}{35}$; $^{-}\frac{3}{5} = ^{-}\frac{21}{35}$

since $^{-}\frac{20}{35} > ^{-}\frac{21}{35}$, $^{-}\frac{4}{7} > ^{-}\frac{3}{5}$

Ex. Order the numbers named: $^{-}\frac{1}{3}, ^{-}\frac{1}{5}, ^{-}\frac{1}{4}, \frac{1}{3}, \frac{1}{6}, ^{-}\frac{1}{2}$

Answer: $^{-}\frac{1}{2} < ^{-}\frac{1}{3} < ^{-}\frac{1}{4} < ^{-}\frac{1}{5} < \frac{1}{6} < \frac{1}{3}$

2545 Properties

This code should be used for all properties of the set of rational numbers. This includes closure, commutativity, associativity, etc.

COMPUTATION

2610 Addition

Ex. Find the sum of $^{-}\frac{4}{5}$ and $^{-}\frac{3}{2}$.

Ex. Find the sum and express in lowest terms: $^{-}15\frac{1}{2} + ^{+}2\frac{3}{8}$.

75

COMPUTATION

Subtraction 2620

Ex. Find the difference: $^{-}3\frac{2}{3} - {}^{+}2\frac{5}{6}$

Ex. Find the difference: $^{+}1\frac{3}{4} - {}^{-}2\frac{1}{2}$

Multiplication 2630

Ex. Multiply: $(^{-}4\frac{2}{7}) \times (3\frac{1}{4})$

Ex. Multiply: $^{-}2.13 \times 1.7$

Division 2640

Ex. Divide: $(^{-}14) \div (\frac{^{-}4}{7})$

Ex. Divide: $\frac{^{-}4}{5} \div \frac{2}{7}$

Sequential operations 2650

Ex. $(\frac{^{-}18}{5} \div \frac{9}{35}) \times (\frac{^{-}3}{7})$ Answer: 6

Ex. $(^{-}10.0956 \div {}^{-}4.7) \div (0.4)$ Answer: 5.37

Several operations in the same lesson 2660

Note: If one or two operations predominate, do not use this code. Code the operations.

TOPIC I: Number Systems

Natural Numbers

Counting Numbers

2720 Definition for a set of natural numbers

The set of natural numbers is shown as $N = \{1, 2, 3, \cdots\}$

2730 Relation to set of whole numbers, nonnegative rationals, integers, negative rationals

$N = \{1, 2, 3, \cdots\}$ set of natural numbers

$W = \{0, 1, 2, \cdots\}$ set of whole numbers

$I = \{\cdots, {}^-2, {}^-1, 0, {}^+1, {}^+2, {}^+3, \cdots\}$

Nonnegative rationals are such fractions as
$\frac{1}{4}, \frac{1}{2}, \frac{3}{8}, \frac{12}{5}, \frac{6}{6}, \frac{0}{9}$, etc.

Negative rationals are such fractions as
$\frac{^-10}{3}, \frac{^-2}{5}, \frac{^-6}{6}$, etc.

The set of natural numbers and the set of whole numbers are subsets of the set of integers.

The set of nonnegative rational numbers and the set of negative rational numbers are subsets of the set of rational numbers.

The rational numbers are a subset of the set of real numbers.

The real numbers are a subset of the set of complex numbers.

Natural Numbers $\subset$ $\underbrace{(\text{Natural Numbers} \cup \text{Zero})}_{\text{Whole Numbers}}$ $\subset$ Integers $\subset$ Rationals $\subset$ Reals $\subset$ Complex Numbers

Natural Numbers (positive integers)
Zero
Negative Integers
→ <u>Integers</u>

<u>Integers</u>
and
Fractional Numbers (positive and negative)
→ <u>Rationals</u>

<u>Rationals</u>
and
Irrationals
→ <u>Reals</u>

<u>Reals</u>
and
Imaginary Numbers
→ <u>Complex Numbers</u>

TOPIC I:

Number Systems

Real Numbers

BASIC CONCEPTS

2750 Irrational numbers developed as nonrepeating decimals

A nonterminating, nonrepeating decimal represents an irrational number. The numeral may or may not have a systematic pattern.

Ex. 1) 0.101001000100001···

2) 0.123456789101112···

ten eleven twelve

3) $\sqrt{2} = 1.41428\cdots$

Examples 1 and 2 have systematic patterns, example 3 does not.

2760 Rational approximations
(See 2870)

Ex. $\sqrt{2}$ is between 1 and 2

$\sqrt{2}$ is between 1.4 and 1.5

$\sqrt{2}$ is between 1.41 and 1.42

$\sqrt{87}$ is between 9 and 10

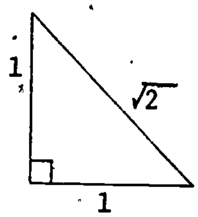
Ex. Using a table of square roots, find a decimal approximation to $\sqrt{43}$.

Ex. $\pi \doteq 3.14$ or $\frac{22}{7}$

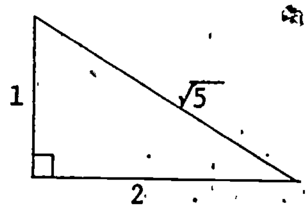
BASIC CONCEPTS

The Pythagorean theorem; construction of line segments with irrational numbers as their lengths 2770
(See 5320, 7155)

Ex. An isosceles right triangle with legs of unit length, has a hypotenuse equal to $\sqrt{2}$.



Ex. Construct a line segment to represent $\sqrt{5}$.



Since $1^2 + 2^2 = 1 + 4 = 5$

Density 2780
(See 1100)

The set of real numbers is *dense*. That is, between any two real numbers there is always another real number.

Ex. Given any two real numbers, a and b, $a < b$, the real number $\frac{a+b}{2}$ is such that $a < \frac{a+b}{2} < b$.

The number line; completeness 2790

There is a one-to-one correspondence between the set of real numbers and the set of points on the number line; i.e., there is exactly one real number corresponding to any given point on the number line, and every real number is a coordinate of some point on the number line. Because of this one-to-one correspondence, we say that the set of real numbers is *complete*.

Real Numbers

BASIC CONCEPTS

2800 Other properties

Ex. For positive numbers a and b,

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

2810 Special irrational numbers: π, e

If c is the circumference and d the diameter of a circle then the ratio $\frac{c}{d}$ is the same for all circles. This special number is called π (pi). π is an irrational number.

Rational numbers often used as approximations to π are 3.1416, 3.142, 3.14, $\frac{22}{7}$.

π is the only special irrational number likely to be encountered in textbooks for grades K-8. Another special irrational number is e which is approximately 2.718. It is the base of the system of natural logarithms.

COMPUTATION

2830 Addition

Ex. $(3 + \sqrt{2}) + (^-5 + \sqrt{2}) = {}^-2 + 2\sqrt{2}$

2840 Subtraction

Ex. $(3 + \sqrt{2}) - (5 - \sqrt{2}) = {}^-2 + 2\sqrt{2}$

2850 Multiplication

Ex. $(3 + \sqrt{2})(2 - \sqrt{3}) = 6 - 3\sqrt{3} + 2\sqrt{2} - \sqrt{6}$

COMPUTATION

Division 2860

Ex. $3 \div \sqrt{2} = \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \doteq \frac{3\sqrt{2}}{2}$

Powers and roots 2870
(See 2760, 0700)

Ex. Find $\sqrt{91}$ to the tenths place. Answer: 9.5

Ex. Find an approximation to the nearest hundredth of $\sqrt{623}$. Answer: 24.96

Use this code when computing roots. If the relationship between rational and irrational numbers is being stressed, code 2760.

Sequential operations 2880

TOPIC I:

Number Systems

Complex Numbers

2910 Development

Complex numbers are numbers of the forms $a + bi$ where a and b are real numbers and i has the property that $i^2 = {}^{-}1$. They are often written as an ordered pair (a, b).

2920 Computation

Ex. $(3 + 2i) + (7 - i) = 10 + i$

Ex. $(3 + 2i)(5 + 4i) = 15 + 22i + 8i^2$

$= 15 + 22i - 8$

$= 7 + 22i$

Difference between number and numeral 3000

A number is the property shared by a collection of matched sets, as 2 is the cardinal number of the sets {*X*, *Y*} and {*A*, *B*}. Numerals are the names for numbers: 5, V, 100, etc. A number is an idea, is abstract and cannot be written or seen. A numeral is a *symbol* for the number, is concrete and can be written and seen.

Different numerals for the same number (renaming)

A. Expanded notation for whole numbers 3010
(See 0200, 0210, 0227, 0229, 0330, 3070, 3080)

Expanded notation is notation using numerals showing the place value of each digit.

Ex. $874 = 800 + 70 + 4$

$874 = 8$ hundreds + 7 tens + 4 ones.

Polynomial form

$(8 \times 100) + (7 \times 10) + (4 \times 1)$

or

$(8 \times 10^2) + (7 \times 10^1) + (4 \times 10^0)$

B. Expanded notation for nonnegative rationals (fractions) 3015

Ex. $3\frac{1}{2} = 3 + \frac{1}{2}$

$.35 = .3 + .05$

$.35 = \frac{3}{10} + \frac{5}{100}$

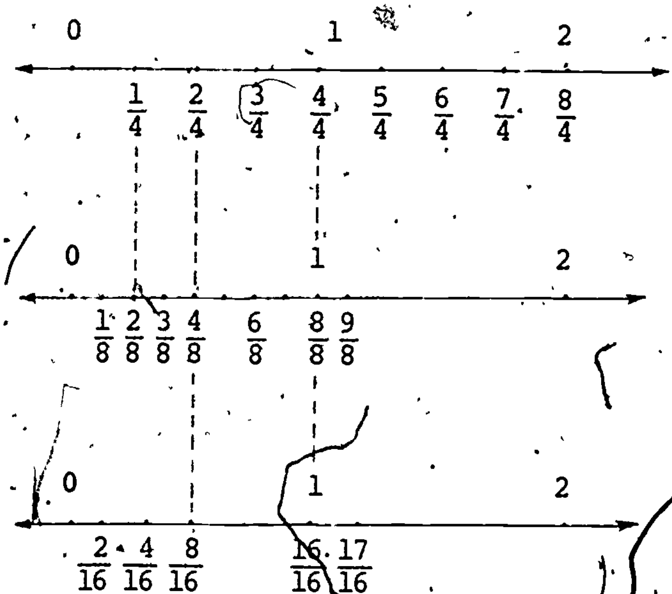
$$.35 = [(3 \times \frac{1}{10^1}) + (5 \times \frac{1}{10^2})]$$

$$56.63 = 50 + 6 + .6 + .03$$

$$56.63 = [(5 \times 10) + (6 \times 1) + (6 \times \frac{1}{10}) + (3 \times \frac{1}{100})]$$

3020 C. Equivalent common fraction notation
(See 1060)

Ex.



The fraction (numeral) $\frac{2}{4}$ may be renamed as $\frac{4}{8}$, $\frac{8}{16}$.

$\frac{1}{2}$ may be renamed as $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, etc.

One may be renamed as $\frac{4}{4}$, $\frac{8}{8}$, $\frac{16}{16}$, etc.

3025 D. Equivalent mixed numeral notation

Ex. $1\frac{3}{4} = \frac{7}{4}$, $4\frac{1}{3} = \frac{13}{3}$, $\frac{14}{3} = 4\frac{2}{3}$

E. Equivalent decimal fraction notation with terminating decimals 3030

A terminating decimal has a finite number of digits.

Ex. $.75 = \frac{3}{4}$

$\frac{3}{8} = .375 = \frac{375}{1000}$

$\frac{1}{2} = .5$

$3\frac{1}{2} = 3.5$

F. Equivalent decimal notation with repeating decimals 3033

A repeating decimal numeral has an initial pattern of digits followed by a continuous repetition of a single digit or a pattern of digits.

Ex. $\frac{1}{6} = .1666\cdots$. $\frac{5}{12} = .4166\cdots$. $\frac{1}{7} = .142857142857\cdots$.

G. Equivalent per cent notation 3035
(See 8004)

Ex. $25\% = .25 = \frac{25}{100} = \frac{1}{4}$

Ex. $\frac{3}{8} = .375 = 37\frac{1}{2}\%$

Ex. $2.5 = 2\frac{1}{2} = 250\%$

H. Other names for a number 3040
(See 1040, 1060)

Use this code largely for other names for natural numbers, whole numbers or integers. Use 3020, 3025, 3030, 3033 and 3035 for other names for rational numbers.

Use when not classified in 3010-3035.

If the purpose of the lesson is the development of basic facts, do not code 3040.

Ex. Some other names for 6 are:

2 × 3	12 ÷ 2
2 + 4	1 + 1 + 4
7 - 1	½ of 12

Place value in base ten

3050 A. Reading and/or writing numerals

Ex. Have the children write the numeral in the air, then trace the dotted numeral in the book.

3060 B. One digit numerals
(See 0020, 0030)

Ex. "What is the largest number (base ten) that can be expressed with a one digit numeral?"

3070 C. Two digit numerals
(See 0035, 3010)

Ex. 12 means 1 ten and 2 ones.

Use code 3070 when place value is being emphasized.

3080 D. Three or more digit numerals
(See 3010)

Ex. 103 means 1 hundred, no tens, and 3 ones.

Use code 3080 when place value is being emphasized.

3090 E. Commas to separate into periods

Ex.

millions	thousands	units or ones
3,	210,	000

F. Rounding numbers 3100
(See 8150)

Ex. 37 rounded to the nearest 10 is 40.
673 rounded to the nearest 100 is 700.
428 rounded to the nearest 100 is 400.

35 may be rounded to the nearest 10 as 30 or 40. The text used will determine the policy.

G. Decimal fractions 3110

Decimal fractions may be considered another way of naming rational numbers which in fraction form have some power of 10 as a denominator.

Ex. $\frac{7}{10} = .7$ $\frac{23}{1000} = .023$

Ex.

	···1	2	3	.	4	5	6	7	8···	
etc.	hundreds	tens	ones		tenths	hundredths	thousandths	ten thousandths	hundred thousandths	etc.

3120 H. Exponential notation
(See 0520, 0670, 0700)

Exponential notation is notation using numerals which have exponents, small numerals written to the right and above a base numeral. An exponent may be either positive or negative. It may be a fraction or zero.

Ex. $874 = (8 \times 10^2) + (7 \times 10^1) + (4 \times 10^0)$

$= (8 \times 10 \times 10) + (7 \times 10) + (4 \times 1)$

Ex. $3^4 = 3 \times 3 \times 3 \times 3$

Ex. $3^{-2} = \frac{1}{3^2} = \frac{1}{3 \times 3}$

Also code such examples as $\left(\frac{2}{5}\right)^2 = \frac{2}{5} \times \frac{2}{5}$.

Use code 3120 when notation of exponents is developed. Operations with ~~exponential notation are coded 0520,~~ 0670.

3130 I. Scientific notation

Scientific notation is of the form 2.69×10^3. For any numeral the decimal point is placed immediately to the right of the first nonzero digit and the number is then multiplied by the integral power of 10 that would have the effect of shifting the decimal point to its original position.

Ex. 25 is written in scientific notation as 2.5×10^1

Ex. $236 = 2.36 \times 10^2$

Ex. $.000236 = 2.36 \times 10^{-4}$

3140 Historical development of number concepts

In primitive times man may have noted the number of animals he killed by dropping a stone on a pile or making a mark for each on a rock realizing that he had more than one animal and later realizing that he needed names for this counting. (See any history of mathematics text for detailed development.)

Historical systems of notation 3150
(See 3160)

A. Egyptian 3151

Ex. 𓍢 in Egyptian symbols means 100 in Hindu-Arabic symbols.

B. Roman 3153

Ex. XV in Roman numerals means 15 in Hindu-Arabic symbols.

C. Other 3158

This includes Babylonian, Mayan, Greek systems

Ex. /// often marked on the rocks or sand means 3 in Hindu-Arabic symbols.

Nondecimal place value systems (other number bases)
(See 3150)

A. Development of place value—other number bases 3160

Nondecimal place value numeration systems are built on bases other than 10 but still use place value.

Ex. 413_{five} or 413 (base five) means
$(4 \times 5^2) + (1 \times 5^1) + (3 \times 5^0)$.

Ex. 413_{eight} or 413 (base eight) means
$(4 \times 8^2) + (1 \times 8^1) + (3 \times 8^0)$.

Ex. 413 in any base five or larger means
$4 \times \text{base}^2 + 1 \times \text{base}^1 + 3 \times \text{base}^0$.

Note: Since 4 is used in the numeral the base must be at least as large as five.

3163 B. Expanded notation

Ex. Write the compact base-seven numeral for
$(1 \times 7^4) + (6 \times 7^3) + (4 \times 7^2) + (3 \times 7) + 2$

Answer: 16432_{seven}

Ex. Write a base-seven name for 6 sets of seven and 4

Answer: 64_{seven}

Ex. $53_{seven} = 40_{seven} + 13_{seven}$

3164 C. Conversion

Ex. Write the base-ten numeral for 312_{four}

$$
\begin{aligned}
312_{four} &= 3\ (4^2) + 1\ (4) + 2 \\
&= 3\ (16) + 4 + 2 \\
&= 48 + 4 + 2 \\
&= 54
\end{aligned}
$$

Ex. Write 54 in base four

$$
\begin{array}{l}
4\overline{)54} \\
\quad 4\overline{)13} \quad R2 \\
\qquad 3 \quad R1
\end{array}
$$

$54 = 312_{four}$

Ex. Write 3075 in base eight

$8^1 = 8$
$8^2 = 64$
$8^3 = 512$
$8^4 = 4096$

```
512)3075(6
    3072
  64)  3(0
       0
   8)  3(0
       0
   1)  3(3
       3
```

$3075 = 6003_{eight}$

D. Computation 3168

Base seven

$$\begin{array}{r} 41 \\ +\ 24 \\ \hline 65 \end{array} \qquad \begin{array}{r} 6 \\ \times\ 5 \\ \hline 42 \end{array}$$

Computations are sometimes done by working with or without a table. Sometimes the problem is converted to base ten and the answer converted to the required base.

E. Fractions 3171

Ex. In base ten .23 represents $\frac{2}{10} + \frac{3}{10^2} = \frac{2}{10} + \frac{3}{100}$.

Ex. In base four .23 represents $\frac{2}{4} + \frac{3}{4^2} = \frac{2}{4} + \frac{3}{16}$.

TOPIC III

Sets

3994 Description of sets

A set is a well-defined collection; that is, one is able to tell whether an object is a distinct member of the set.

A set is usually indicated by braces {xx} or closed curves

xx , $\boxed{xx}$.

Ex. Physical objects - a set of dishes
a set of dominoes

Abstract - the set of whole numbers

4000 Set members or elements

Members - each object in the set (collection) is a member or element of the set.

Ex. The set {□ ○ ◁ ▭} has 4 members or elements.

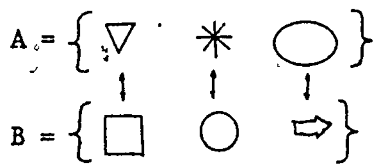
The square, circle, triangle, rectangle are elements of the set.

In the set {6, 7, 8, 9} each number is a member or an element of the set.

KINDS OF SETS

4010 Equivalent sets (one-to-one correspondence)
(See 0050, 0060, 0100)

Equivalent sets have the same number of members but not necessarily the identical members. Members of equivalent sets can be paired in one-to-one correspondence.

Ex. 

A and B are equivalent sets since each has 3 elements or since the elements can be shown in one-to-one correspondence.

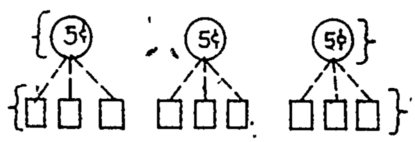
Non-equivalent sets (general) 4030
(See 0100)

Non-equivalent sets do not have the same number of members and cannot be paired in one-to-one correspondence.

Ex. {□ ○} and {▽ □ ⬡} are *not* equivalent sets.

Non-equivalent sets (one-to-many correspondence) 4035

Ex. If one nickel will buy 3 pieces of candy, then two nickels will buy 6 pieces of candy, etc.



If the drawing one element from the set of nickels is matched with three elements from the set of pieces of candy.

Equal sets (identical) 4037

Equal sets have exactly the same members. They will then have the same number of members and are therefore also equivalent.

Ex. {□ ○ ◺} and {○ ◺ □} are equal sets since the members or elements are identical though not shown in the same order.

4040 **Unequal sets**

Unequal sets do not have identical members or elements though they may have the same number of elements in which case they are equivalent sets.

Ex. See 4010

{□ ○ △} and {◇ ▱ ○} are unequal since elements are not identical.

{1,2,3,4,5} and {12,3,4,5} are unequal.

4060 **Subsets**
(See 0250, 0540)

If each member of set B is a member of a set A, we say that B is a subset of A.

Ex. A = Set of all pupils, boys and girls, in the room

B = Set of all boys in the room

B is a subset of A since all the boys belong to the set of all the pupils.

Ex. N = {2,4,6}. The subsets of N are {2}; {4}; {6}; {2,4}; {2,6}; {4,6}; { }; {2,4,6}. The symbol used to indicate a subset is $\subset$. $B \subset A$ is read B is a subset of A.

Note: A subset may be removed from a set. See 0250. A set may be partitioned into equivalent subsets (See 0540) or non-equivalent subsets.

4070 **The empty set**
(See 0020)

The empty set has no members or elements. The set of students with four legs is an empty set. The cardinal number of the empty set is zero. { } is one symbol for the empty set. The empty set is a subset of every set.

Disjoint sets 4090

Disjoint sets are sets which have no elements in common.

Ex. A = {○ □} B = {△ ▭} A and B are disjoint sets.

$A \cap B = \{\ \}$

Ex. C = {1,3,0} D = {7,4}

$C \cap D = \{\ \}$ or $\emptyset$

Union of sets 4093
(See 0120)

The union of two sets, denoted by the symbol $\cup$, is the set of all elements belonging to either of the two sets or to both of them. Elements common to both sets are not repeated when naming the set union.

Ex. A = {1,3,5} B = {2,3,5,7} $A \cup B$ = {1,2,3,5,7}

Intersection of sets 4095

The intersection of ~~two~~ or more sets, denoted by the symbol $\cap$, is the set of elements common to both or all sets.

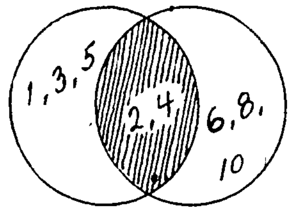
Ex. If A = {1,3,5,7} and B = {5,7,9,11} then A and B are intersecting sets.

$A \cap B$ = {5,7}; read A intersection B, is the set {5,7} where 5 and 7 are the elements common to both sets.

Venn diagrams 4097
(See 8135)

Venn diagrams are diagrams which use overlapping or intersecting circles to show relationships between sets.

Sets

Ex. A  B

$A = \{1,2,3,4,5\}$ $B = \{2,4,6,8,10\}$

$A \cap B = \{2,4\}$ The shaded area shows the intersection.

Use 4097 only if Venn diagrams are identified by the authors.

For use of Venn diagrams in logic, code 8135.

4100 Finite sets

Finite sets are sets which can be defined by counting, with the counting coming to an end. There is a whole number that identifies the number of members.

Ex. $A = \{2,4,6\}$ all the members are identified.

$B = \{2,4,6,\cdots,20\}$ is a finite set since all the members can be identified by counting.

4110 Infinite sets

Infinite sets are those which cannot be named by counting, with the counting coming to an end.

Ex. $\{1,2,3,4,\cdots\}$ The set of natural numbers is an infinite set.

The three dots indicate that the set is infinite and that you may write other elements which continue the pattern indefinitely.

4120 Universal set, difference and complement

A universal set is the set containing all elements under consideration and is usually designated by U.

Ex. U = {all states in the U.S.}

A = {Iowa, Minnesota, New York} A is a subset of U.

The set difference of A and B (denoted by A/B or A - B) is the set of elements that are in A but not in B.

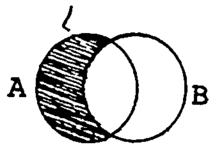
Ex. A = {January, February, March}

B = {March, April, May, June}

A - B = {January, February}

B - A = {April, May, June}

Ex.



A - B is represented by the shaded portion of the diagram.

If U is the universe under consideration, and A any subset of U, then the set composed of all the elements of U that are *not in* A is called the complement of A, and is usually designated by A'.

Ex. If U represents all the pupils in a room and A represents all the pupils with blue eyes, then the set complement of set A is A' or all the pupils who do do *not* have blue eyes.

Solution sets and replacement sets 4125
(See 8170)

The set of elements which when used to replace the variable(s) in an open sentence make it a true sentence, or the set of all numbers that are solutions for a number sentence, is called a solution set.

Ex. $3 \times n > 60$. The solution set for n, an integer, is $\{21,22,23,\cdots\}$

If $A = \{x|x > 5\}$ in the universe of whole numbers, then the solution set $S = \{6,7,8,\cdots\}$

4160 Cartesian product sets (cross products)
(See 0390)

The Cartesian product of A and B is the set of all pairs of elements from set A and set B such that the first element in the pair is from set A and the second from set B.

Ex. $A = \{x,y\}$, $B = \{1,2,3\}$. The product set is $\{(x,1), (x,2), (x,3), (y,1), (y,2), (y,3)\}$. The number of pairs in the product set is the cardinal number of the Cartesian product. In this case it is 6.

Note: See 0390 for use with introduction of multiplication of whole numbers.

TOPIC IV:

Geometry

INTUITIVE CONCEPTS OF GEOMETRIC FIGURES AND IDEAS

Geometric figures in environment 5010

Shapes such as circles, squares and triangles become familiar through pictures or objects seen in the room, on trips, at home.

Note: Use this code in introductory lessons on geometric figures if a variety of familiar shapes is used.

Geometric designs or patterns (sequences) 5020

Ex. □—○□—○— ? — ?

Continue the pattern.

Spatial relations without measurement (size, position, betweenness) 5030

Ex. Mark an *x* on the larger ball.
Mark an *x* on the largest ball.
Mark an *x* above (below) the doll.
Mark an *x* to the left (right) of the doll.

Two dimensional figures (plane) 5040

Plane figures are two dimensional.

Ex. Some models of plane figures are the surfaces of floors, window panes and doors.

Three dimensional figures (solid) 5050

Figures in space are three dimensional.

Ex. Some models of three dimensional figures are a cereal box, a tin can and an ice cream cone.

Geometry

5060 Curves: simple, non-simple; closed, open
(See 5174)

A simple curve can be traced without passing through any point twice.

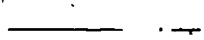
A closed curve is undefined but may be thought of as a set of points represented by a drawing beginning and ending at the same point.

Line segments are considered to be curves.

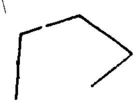
Ex. Pictures of curves:

A. simple, open

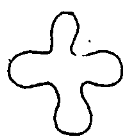
B. non-simple, closed



C. simple, open

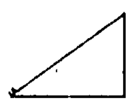


D. simple, open

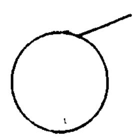


E. simple, closed

F. non-simple, open



G. simple, closed



H. non-simple, open

Regions formed by simple closed curves 5070

A mathematical region is a particular multipoint space set in which any two points can be connected by a continuous curve without passing through any point which is not in the particular multipoint space set.

A simple closed curve separates the plane into three sets of points:

All points outside the curve (exterior region).
All points inside the curve (interior region).
All points of the curve itself (sometimes called *the boundary*). The boundary has no points in common with either its interior or exterior regions.

A region is *closed* if it contains all of its boundary points. A region is *open* if it contains none of its boundary points. A region is neither open nor closed if it contains at least one but not all of its boundary points.

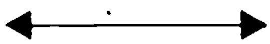
Simple closed surfaces 5075

A simple closed surface can be thought of as the space counterpart of a simple closed curve.

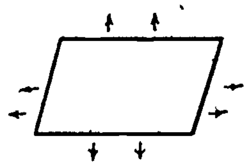
Examples of simple closed surfaces are spheres, polyhedrons, cylinders.

Representations of point, line, plane, space 5080

A point is a concept which, like a number, exists only in the mind. As a numeral represents a number, a dot (.) represents a point. The tip of a pen or the sharp end of the lead in a pencil would suggest a point.

 A line is a set of points extending infinitely in opposite directions.

Ex. The intersections of walls and a ceiling suggest lines although they do not go on infinitely far.



A flat surface suggests a plane. The set of points in a plane extends infinitely in all directions.

Ex. Some things in the room which suggest a plane are a desk top, floor, walls or any flat surface.

Space is the set of all points. A book, ball or box of dominoes suggest space figures (three dimensional figures).

5081 Optical illusions

CONCEPTS OF GEOMETRIC FIGURES AND IDEAS EXPLORED IN DEPTH

5090 Point

A point is undefined. It has no measure. It is a zero dimensional concept. The intersection of two lines is a point. Between any two distinct points in space there is always another point.

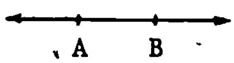
5100 Line

A line is undefined. It is a one dimensional concept. Its measure is length. Every point on a line is between two other points. For any two points there is only one line which contains both of them. Two distinct lines intersect in at most one point.

5101 Line segment
(See 5600)

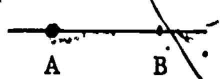
A line segment is the union of two points on a line and all the points on the line between them.

Ex. Pictures of line segments:

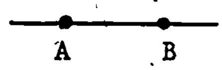


The line segment consists of point A, point B and all points between them. A and B are called the endpoints of the line segment.

Note: A half open line segment is the union of the points between A and B and one of the endpoints.



An open line segment is the set of all points between two given points. (It does not contain the given points.)



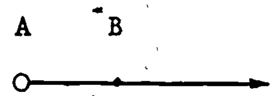
Ray 5103

A ray is defined as an infinite set of points (a subset of a line) with only one endpoint. A second point in the ray helps to name it. A half-line may be thought of as a ray without its endpoint.

A ray is the union of a half-line and an endpoint.

Ex. 

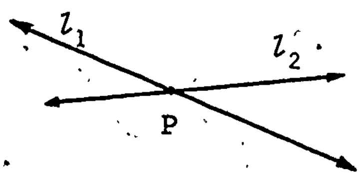
This picture represents a ray. (Note solid dot at A.)

Ex. 

This picture represents a half-line. (Note open dot at A.)

Related lines: intersecting, parallel, skew, oblique, ... 5105

Lines drawn through a common point are called intersecting lines.

Ex. 

l_1 and l_2 are intersecting lines throught point P. How many lines can intersect at point P? (An infinite number.)

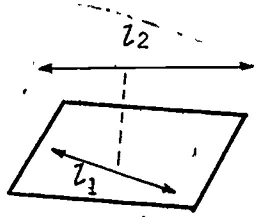
Parallel lines are lines in the same plane having no points in common. Their intersection is the empty set.

Ex. Two rails of a railroad track suggest parallel lines.

Skew lines are lines that have no point in common and do not lie in the same plane.

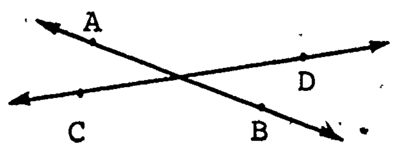
Geometry

Ex. A string stretched across the floor and a string stretched by two pupils at waist height so that it is not parallel to the first string represent skew lines.



Oblique lines are two lines in a plane which are neither parallel nor perpendicular. They intersect to form pairs of obtuse angles and pairs of acute angles.

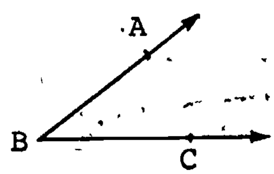
Ex.



5115 Angles

An *angle* is the union of two noncollinear rays with a common endpoint.

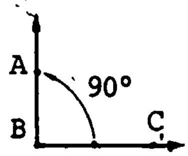
Ex.



Kinds of angles 5125

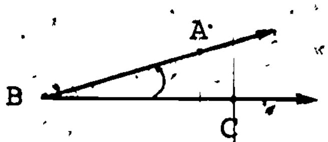
A *right angle* is an angle whose degree measure is 90.

Ex.



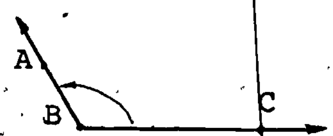
An *acute angle* is an angle whose degree measure is greater than 0 and less than 90.

Ex.



An *obtuse angle* is one whose degree measure is greater than 90 and less than 180.

Ex.



Regions formed by angles 5140

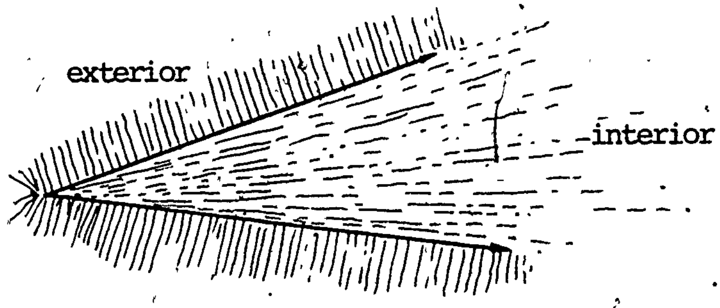
An angle separates a plane into three distinct sets of points:

the set of points between the rays (*interior region*).

the set of points of the angle (*boundary*).

the set of points not between the rays and not on the boundary (*exterior region*).

Ex.



Code regions formed by open curves here, except see 5510 for regions formed by a line.

See also 5070, 5510.

5143 Planes

Every plane contains at least three points not in a straight line (not collinear). A plane is a surface such that a straight line joining any two of its points lies entirely in the surface. A plane is a flat surface. The intersection of two planes is a line. An unlimited number of planes can pass through a line determined by two points. A plane figure has two dimensions, length and width.

Polygons (plane figures)

5145 A. General properties of polygons

A polygon is a simple closed curve (see 5060) formed by the union of line segments.

Use this code for the properties of polygons in general, such as number of vertices, number of diagonals or when more than two types of polygons are considered in the same lesson.

5148 B. Relationship of angles or sides of a polygon

Ex. The sum of the degree measure of the angles of a triangle is 180.

Ex. The sum of the lengths of any two sides of a triangle is greater than the length of the remaining side.

Ex. The longest side of a triangle is opposite the greatest angle.

Note: The Pythagorean theorem and its use should be coded 5320 or 2770.

C. Triangles 5150

See 5280 and 5290 for perimeter and area.

A *triangle* is a polygon of three sides.

An *equilateral* triangle is a triangle whose three sides are equal in length (or measure).

An *isosceles* triangle is a triangle with at least two sides equal in measure.

A *scalene* triangle is a triangle with no two sides equal in measure.

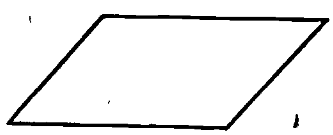
A *right* triangle is a triangle having one angle whose degree measure is 90.

D. Quadrilaterals 5160

Quadrilaterals are polygons having four sides.

A *parallelogram* is a quadrilateral with both pairs opposite sides parallel.

Ex.

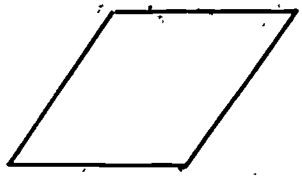


A *rectangle* is a parallelogram with one right angle (and therefore with four right angles).

A *square* is a rectangle with two adjacent sides equal in measure (and therefore with all four sides equal in measure).

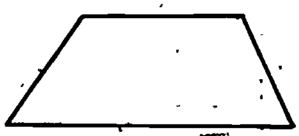
A *rhombus* is a parallelogram with two adjacent sides equal in measure (and therefore with all four sides equal in measure).

Ex.



A *trapezoid* is a quadrilateral with one and only one pair of parallel sides.

Ex.



5170 E. Other polygons

A *pentagon* is a polygon with five sides.

A *hexagon* is a polygon with six sides.

An *octagon* is a polygon with eight sides.

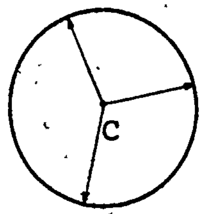
5174 Topological concepts
(See 5060)

Use this code for intensive work with curves, the Möbius strip, Euler's rule for the edges, vertices and faces of simple closed surfaces and other topics from topology.

5180 Circles

A *circle* is a set of all points in a plane which are a given distance (radius) from a given point called the center of the circle.

Ex.



All the points on the curve are equidistant from the center C.

The radius, diameter and a chord of a circle have special characteristics.

A circle determines three sets of points in the plane: interior and exterior regions and the circle itself.

Central angles cut off arcs and sectors of circles.

See codes 5280 and 5290 for coding circumference and area of circles.

Three dimensional space 5183

Space is the set of all points.

A closed three dimensional figure separates space into three sets of points:

All points outside the figure (*exterior region*).

All points inside the figure (*interior region*).

The points of the figure itself (*boundary*).

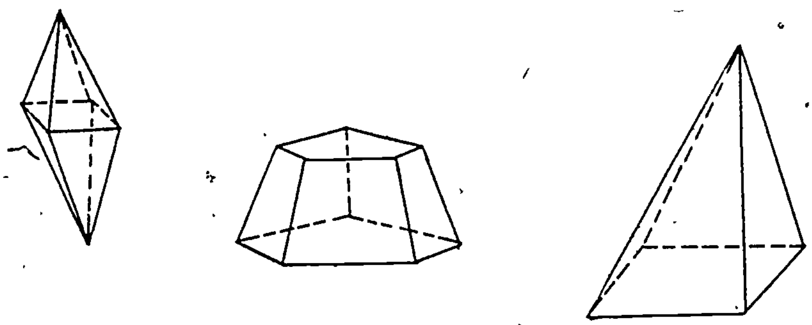
Ex. The interior region of a sphere is the union of its center point and all points whose distances from the center point are less than the radius. The exterior region is all other points not on the surface of the sphere.

Three dimensional figures

A. General properties of three dimensional figures 5185

A polyhedron is the union of closed polygonal regions which bound a portion of space.

Ex.



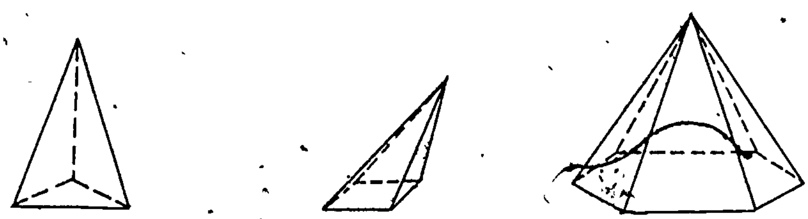
The polygonal regions are called *faces*.
The faces intersect in *edges* (line segments).
The edges intersect in *vertices* (points).

Some other three dimensional figures are cones, spheres, cylinders and prisms.

Use this code if more than two types are considered in the same lesson.

5186 B. Pyramid

A pyramid is a polyhedron formed by the union of a polygonal region (called the *base*) and triangular regions (called the *lateral faces*). If the base is a triangle, the pyramid is a triangular pyramid. If the base is a square, the pyramid is a square pyramid.



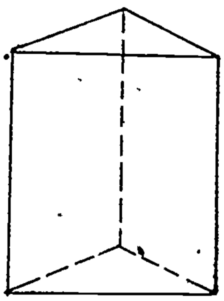
triangular pyramid   square pyramid   hexagonal pyramid

C. Prism

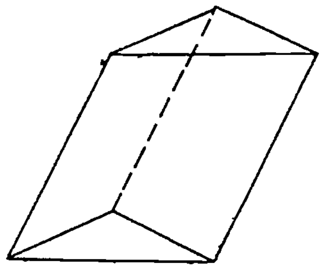
A prism is a polyhedron with two faces (called *bases*) that are congruent polygonal regions in parallel planes and other faces (called *lateral faces*) which are regions bounded by parallelograms.

If the lateral edges are perpendicular to the base, the prism is called a *right prism*.

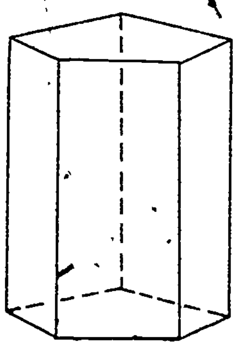
Prisms are named according to the number of sides in their bases.



right triangular prism

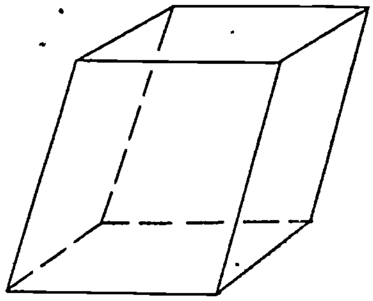


oblique triangular prism



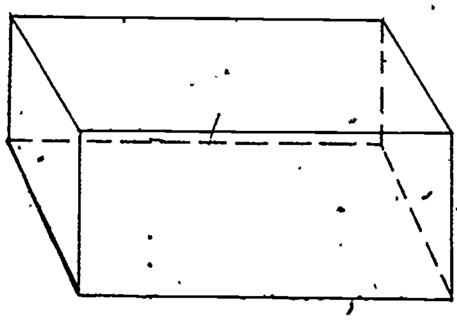
right pentagonal prism

If all of the faces of a prism are parallelograms, then it is called a *parallelepiped*.



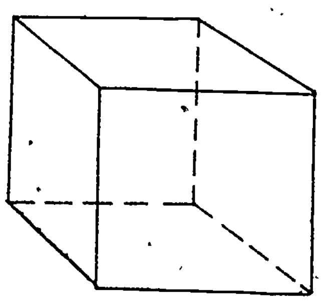
parallelepiped

If all of the faces of a prism are rectangles, then it is called a *rectangular solid.*



rectangular solid

If all of the faces of a prism are *squares*, then it is called a cube.

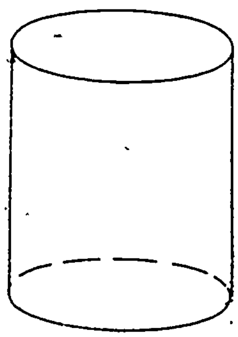


cube

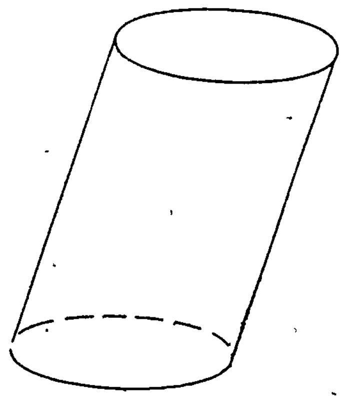
D. Cylinder 5191

A *circular cylinder* is formed by the union of two congruent circular regions (*bases*) in parallel planes and the line segments joining corresponding points of the circles.

If the line joining the centers of the circular regions is perpendicular to the bases, the cylinder is called a *right circular cylinder*.



right circular cylinder

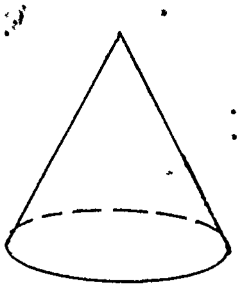


oblique circular cylinder

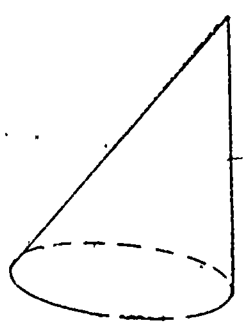
E. Cone 5192

A circular cone is the union of a circular region (*base*) and all line segments joining a point (*vertex*) not in the plane of the base to points on the circle.

If the line joining the vertex to the center of the base is perpendicular to the plane of the base, the cone is called a *right cone*.



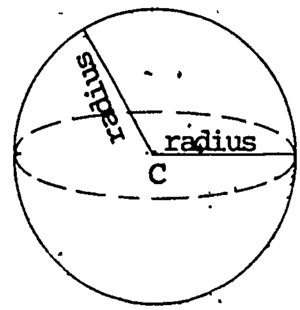
right circular cone



oblique circular cone

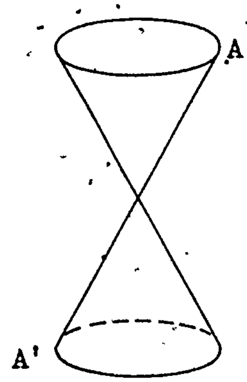
5194 F. Sphere

A *sphere* is the set of all points in space that are a given distance (the *radius*) from a given point called the center.



5195 Conic sections: the ellipse, circle, parabola and hyperbola

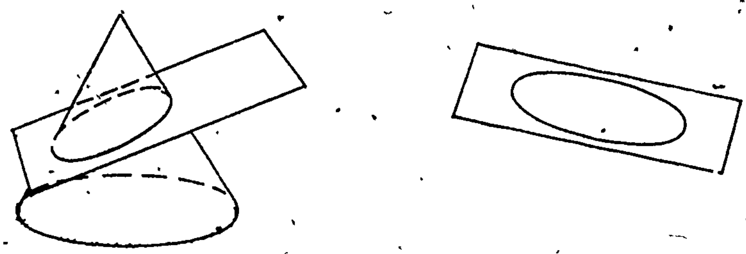
Conic sections, or *conics*, are curves which can be formed by the intersection of a plane and a right circular conical surface.



A conical surface is of unlimited extent and has two parts called *nappes*. Lines on the surface (such as AA') are called *elements*.

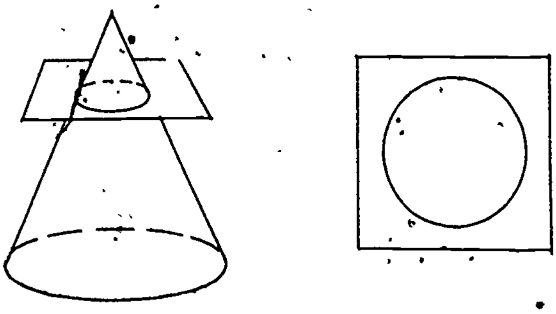
The shape of a conic depends on the position of the plane with respect to the elements on the conical surface.

Ex. An *ellipse* is formed by a plane which intersects only one nappe of the surface and cuts all of the elements.



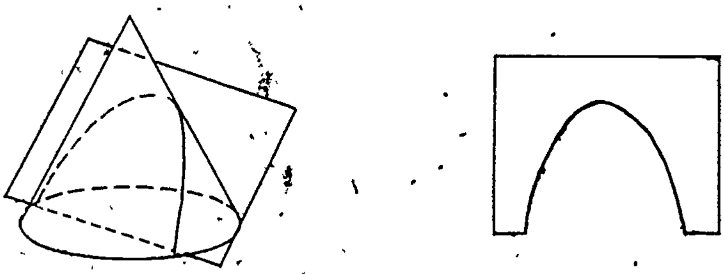
One equation of an ellipse is: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

A *circle* can be regarded as a special case of an ellipse.



One equation of a circle is: $x^2 + y^2 = r^2$.

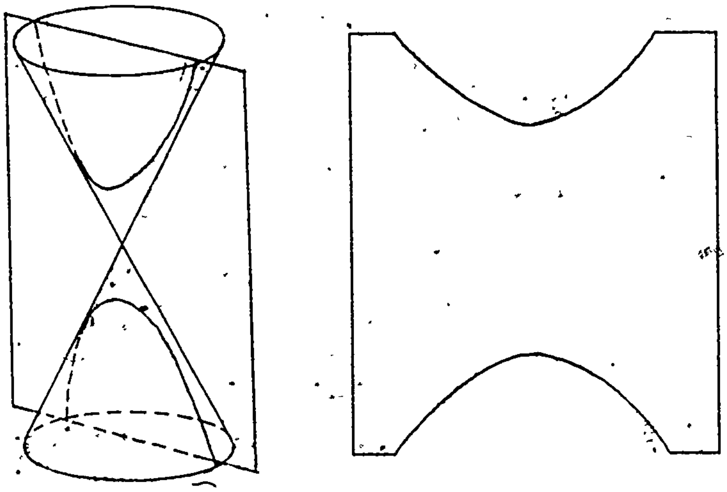
Ex. A *parabola* is formed when the plane is parallel to an element.



One equation of a parabola is $y^2 = 2px$.

Ex. A *hyperbola* is formed when the plane cuts both nappes of the cone.

Geometry



One equation of a hyperbola is: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$

CONSTRUCTIONS

Only recognized geometric constructions will be coded 5210, 5220 and 5230. Drawing geometric figures will be coded under 5080.

5210 A. Line constructions (one dimensional figures)

5220 B. Two dimensional figures (plane figures)

5230 C. Three dimensional figures (figures in space)

METRIC GEOMETRY

Comparing sizes, shapes, distances

5240 A. Congruency

Congruency is the property of the relation of two geometric figures having the same size and shape.

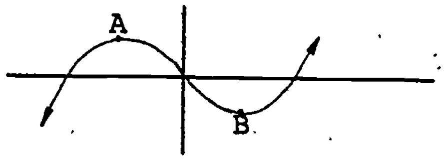
Angles are congruent when their measures are equal.

Two dimensional figures are congruent when their corresponding sides and corresponding angles are equal in measure.

B. Symmetry 5245

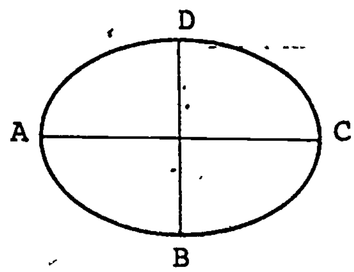
A geometric configuration is said to have symmetry with respect to a point, a line or a plane when for every point on the figure there is another point such that the pair correspond with respect to the point, the line or the plane.

Ex.



Points A and B are symmetric to the origin, a point.

Ex.



The ellipse ABCD is symmetric to the line AC and to the line DB.

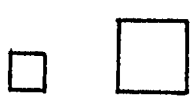
C. Transformations 5248

Ex. Translations, rotations, reflections and inversions are examples of transformations.

D. Similarity 5250

Similar geometric figures have the same shape, but not necessarily the same size.

Ex.



Geometry

Similar polygons have the angles of one equal in measure to the corresponding angles of the other and the measures of the corresponding sides in proportion.

5255. E. Similarity: scale drawing
(See 8000)

In a scale drawing all distances are in the same ratio to the corresponding distances on the original figure.

Measurement of geometric representations

5260 A. Line segments with ruler and/or compass or other measuring device
(See 6030, 6032, 6060, 6065)

5270 B. Angles with protractor and/or compass or other measuring device

5280 C. Perimeter or circumference of simple closed curves
(See 6030, 6032)

5290 D. Area of plane figures
(See 6034, 6035)

Include lessons finding surface areas of solids.

5300 E. Volume of solids
(See 6036, 6037)

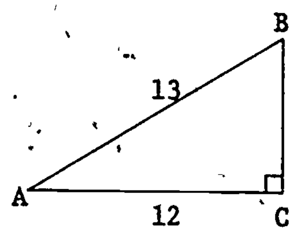
5310 F. Surface area of solids

For materials coded with previous lists, see 5290.

5320 G. The Pythagorean theorem and the distance between two points

The Pythagorean theorem: given a right triangle, the square of length of the hypotenuse (side opposite the right angle) is equal to the sum of the squares of the lengths of the other two sides.

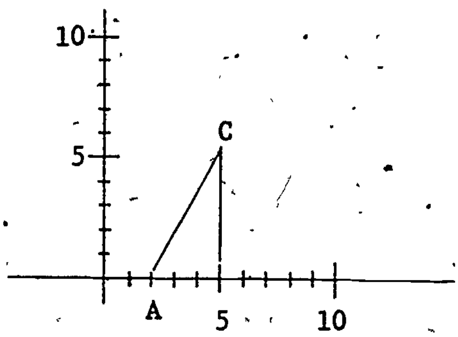
Ex. Given the right triangle ABC (right angle at C)



$$13^2 = 12^2 + 5^2$$

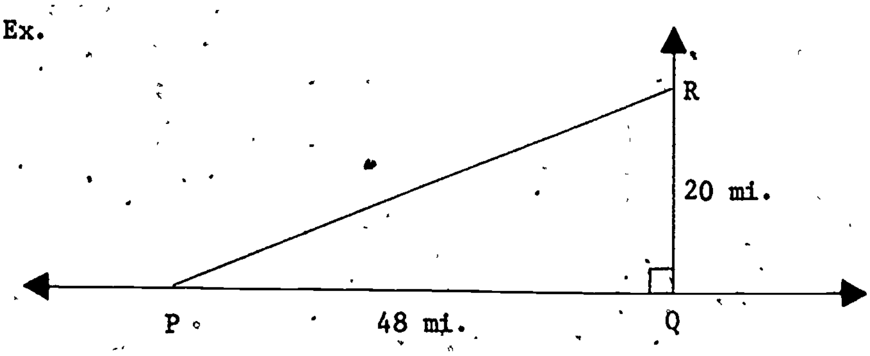
$$169 = 144 + 25$$

Ex.



Find the distance from A to C

Ex.



Find the distance from P to R

Ex. Find the distance between the points whose coordinates are (5,6) and (11,14).

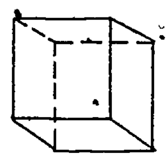
If the emphasis of the lesson is on irrational numbers, code 2770.

OPERATIONS WITH GEOMETRIC FIGURES

5410 Union

Union - See 4093

Ex. The union of six plane rectangular regions forms a rectangular solid.



5420 Intersection

Intersection - See 4095

The faces of a rectangular solid intersect in line segments. The base and the conical surface of a right circular cone intersect in a circle.

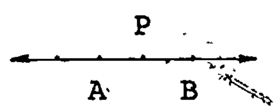
OTHER TOPICS

5510 Separation of sets of points

See 5070 for separation of points of a plane by a *curve*.
See 5140 for separation of points of a plane by an *angle*.

A point separates a line into three distinct sets of points: two half-lines and the point itself.

Ex.



The point P separates the line AB into three distinct sets of points: the half-line PB, the half-line PA and the point P.

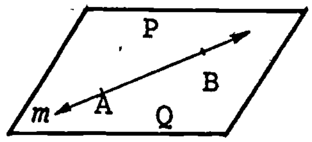
The notation for the half-line PB is PB.

The notation for the half-line PA is PA.

A point is zero dimensional and it separates a line which is one dimensional.

A line separates a plane into three distinct sets of points: two half-planes and the line itself.

Ex.



The line AB separates the plane *m* into three distinct sets of points: the set of points on the line AB, the set of points on the P side of line AB and the set of points on the Q side of line AB.

The notation sometimes used for the half-plane is as follows:

$\overset{P}{\overleftrightarrow{AB}}$ denotes the "P" side of the line AC.

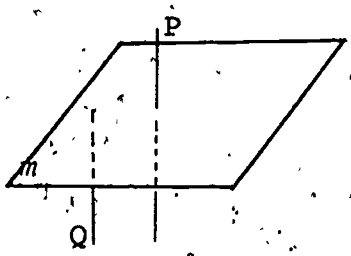
$\overset{Q}{\overleftrightarrow{AB}}$ denotes the "Q" side of the line AB.

The line is one dimensional and it separates a plane which is two dimensional.

See 5183 for separation of points of space by a *three dimensional* figure.

A plane separates space into three distinct sets of points: the two half-spaces and the plane itself.

Ex.



Geometry

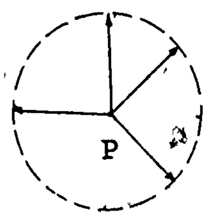
The plane *m* separates space into three distinct sets of points: the half-space on the P side of plane *m*, the half-space on the Q side of plane *m* and the plane *m*.

The plane is two dimensional and it separates space which is three dimensional.

5520 Locus of points

Locus of points is any set of points which satisfy one or more conditions.

Ex. The locus of points in a plane equidistant from a fixed point is a circle.



Conditions:

Points lie in a plane.
There is a fixed point P.
All points in the locus must be equidistant from P.

5600 Geometric notation

Use this code for lessons stressing reading and writing geometric notation.

Ex. Line segment AB is written $\overline{AB}$.

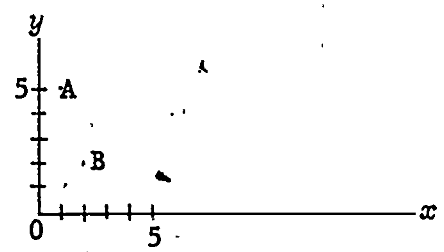
Line AB is written $\overleftrightarrow{AB}$.

Triangle with vertices A,B,C is named Δ ABC.

5700 Vectors

A *vector* is a directed line segment. The vector that begins at point A and ends at point B may be written $\overrightarrow{AB}$. The vector that begins at point B and ends at point A may be written $\overrightarrow{BA}$.

Ex.



$\overrightarrow{AB}$ is the vector from (1,5) to (2,2).

$\overrightarrow{BA}$ is the vector from (2,2) to (1,5).

Non-Euclidean geometrics 5800

A non-Euclidean geometry is (1) a geometry which rejects Euclid's parallel postulate and retains the other postulates, (2) any geometry not based on Euclid's postulates.

TOPIC V

Measurement

6000 Meaning of measurement (direct, indirect)

To measure means to compare an object with some suitable unit.

Ex. Length is measured by a linear unit such as an inch, a yard or a centimetre. Area is measured by a square unit such as a square foot, a square metre or an acre. Weight is measured by a gravitational unit such as a pound or kilogram.

6001 Approximate nature of measurement

No measurement is exact. If you are measuring a line segment your measurement will be affected by the width of the dots at the endpoints, the angle at which you see the lines on your ruler, worn edges of your ruler and so on.

6002 Significant digits

In a measurement, significant digits are those digits needed to name the number of units.

	Measurement	Unit	Nearest No. of Units	No. of Significant Digits
Ex.	506 in.	inch	506	3
		ten inches	51	2
		hundred inches	5	1
Ex.	2.83 ft.	feet	3	1
		tenths of a foot	28	2
		hundredths of a foot	283	3

	Measurement	Unit	Nearest No. of Units	No. of Significant Digits
Ex.	608,000 mi.	thousand miles	608	3
Ex.	.0079 cm.	thousandths of a centimetre	8	1

In scientific notation, all digits in the multiplier of the power of ten are significant.

Ex. 2.080×10^3 has four significant digits.

Precision 6003

The smaller the unit of measure the greater the precision. See 6005.

If the unit of measure is ½ inch and something is measured to the nearest ½ inch, the precision of the measurement is ½ inch.

Round-off error 6004

The greatest possible round-off error is equal to half the place value of the digit to which we are rounding.

Ex. We might know that the population of a city is 238,469. For simplicity we might round this number to 238,000. The difference between 238,469 or 238,000 or 469 is called the round-off error.

Ex. Rounding to tenths:

$37.24 = 37.2$

Actual round-off error: $37.24 - 37.2 = 0.04$

Greatest possible round-off error: $\frac{1}{2} \times 0.1 = 0.05$

The greatest possible error 6005

Measurement

In any measurement the greatest possible error is ½ the smallest division (unit) used on the measuring instrument.

Ex. The greatest possible error in measuring 5 inches with a ruler marked to half inches is ¼ inch.

6006 Relative error

The relative error of a measurement is the ratio of the greatest possible error to the measure.

Ex. The distance between two cities is 500 miles (to the nearest hundred miles). What is the relative error?

Measurement	500 miles
Units of measure	100 miles
Greatest possible error	50 miles (half of the unit)

Relative error $\frac{50}{500} = \frac{1}{10}$ or 10%

UNITS OF MEASURE

Historical development of units of measure

6009 A. Non-standard units such as foot, cubit, furlong leading to the standard English system

6010 B. Metric units

Linear units of measure
(See 6060, 6065)

6028 A. Non-standard

6030 B. English units for yards or less
(See 5260, 5280)

6032 C. Metric units for metres or less
(See 5260, 5280)

Square units of measure in English and non-standard units (See 5290) 6034

Square units of measure in the metric system (See 5290) 6035

Cubic units of measure in the English and non-standard systems (See 5300) 6036

Cubic units of measure in the metric system (See 5300) 6037

Other concepts of measurement and appropriate units 6038

Ex. Decibel, light year, calorie, kilowatt, degrees of latitude and longitude, miles per hour, etc.

Money 6040

Time 6050

Distance in English units for lengths longer than a yard (See 5260) 6060

Distance in metric units for lengths longer than a metre 6065

See 5260 for measurement of line segments.

Liquids in English and non-standard units 6070

Liquids in metric units 6075

Temperature: Fahrenheit and Celsius (centigrade) 6080

Measurement

6090 Weight in English and non-standard units

6095 Weight in metric units

6100 Dry measures

6110 Quantity (dozen, gross, etc.)

6120 Operations related to denominate numbers

Ex.

$$\begin{array}{r} 3 \text{ hr. } 10 \text{ min.} \\ + \underline{2 \text{ hr. } 50 \text{ min.}} \\ 6 \text{ hr.} \end{array}$$

Code 6120 and 6050.

Ex.

$$\begin{array}{r} 6 \text{ lb. } 10 \text{ oz.} \\ - \underline{4 \text{ lb. } 15 \text{ oz.}} \\ 1 \text{ lb. } 11 \text{ oz.} \end{array}$$

Code 6120 and 6090.

6130 Conversion to other standard units measuring several kinds of nongeometric quantities

Ex. In one lesson:

10 pecks $\overset{m}{=}$ 2 bushels 2 pecks

90 minutes $\overset{m}{=}$ 1 hour 30 minutes

15 quarts $\overset{m}{=}$ 3 gallons 3 quarts

etc.

Note: If conversion is being developed with one kind of nongeometric quantity only, code under the quantity.

Ex. 21 days $\stackrel{m}{=}$ 3 weeks

120 minutes $\stackrel{m}{=}$ 2 hours

24 months $\stackrel{m}{=}$ 2 years

3 days $\stackrel{m}{=}$ 72 hours

etc. Code 6050.

Several concepts of measurement in the same lesson 6140

TOPIC VI:

Number Patterns and Relationships

ELEMENTARY NUMBER THEORY

7000 Odd and even numbers

(See 0080)

Even numbers are the integers divisible by 2.

$E = \{\cdots, {}^{-}4, {}^{-}2, 0, {}^{+}2, {}^{+}4, \cdots\}$

Odd numbers are integers not in the set of even numbers.

$O = \{\cdots, {}^{-}3, {}^{-}1, {}^{+}1, {}^{+}3, {}^{+}5, \cdots\}$

(See also 7055)

7010 Factors and primes

In any statement such as $5 \times 8 = 40$, 5 and 8 are called *factors*.

A prime number is a whole number that has only two integral factors, itself and 1.

Ex. 2,3,5,7,11,13,$\cdots$. The number 1 is usually excluded as a prime since it has only one factor.

7020 General composite numbers

A composite number is a whole number which has more than two factors, itself and 1. A natural number greater than 1 which is not a prime number is a composite number.

Ex. 12 is a composite number since its factors are 1, 2, 3, 4, 6 and 12.

ELEMENTARY NUMBER THEORY

Special composite numbers 7030

A perfect number is an integer which is equal to the sum of all of its factors excluding itself.

Ex. 6 = 1 + 2 + 3; 28 = 1 + 2 + 4 + 7 + 14

Relatively prime numbers have no factor except unity in common.

Ex. 8 = 1 × 2 × 2 × 2

9 = 1 × 3 × 3

Both 8 and 9 are composite numbers but they are relatively prime to each other.

Ex. 3 and 5 are relatively prime.

Numbers are amicable numbers if the sum of the factors of each number (excluding itself) equals the other number.

Ex. 220 and 284

The factors of 220 are 1,2,4,5,10,11,20,22,44,55,110; their sum is <u>284</u>.

The factors of 284 are 1,2,4,71,142; their sum is <u>220</u>.

Greatest common factor 7050

The greatest number which is a factor of each of two or more natural numbers is their *greatest common factor*. This number is also called the greatest common divisor.

Ex. 4 is the GCF of 8 and 12.

Euclidean algorithm 7051

The Euclidean algorithm is a method of finding the greatest common factor of two numbers.

See 7050

ELEMENTARY NUMBER THEORY

The larger number is divided by the smaller. The divisor is divided by the new remainder. The process is continued until the remainder is zero. The final division is the greatest common factor.

Ex. Find the GCF of 368 and 80

$$\begin{array}{r} 4 \\ 80\overline{)368} \end{array} \quad R\ 48$$

$$\begin{array}{r} 1 \\ 48\overline{)80} \end{array} \quad R\ 32$$

$$\begin{array}{r} 1 \\ 32\overline{)48} \end{array} \quad R\ 16$$

$$\begin{array}{r} 2 \\ 16\overline{)32} \end{array} \quad R\ 0$$

The GCF of 368 and 80 is 16.

7055 Multiples
(See 0080)

Multiples of a number N are numbers (products) obtained by multiplying N by integers.

Ex. 10, 35, 125 and 5000 are multiples of 5.

Ex. The set of multiples of 2 is
$\{\ldots, {}^-4, {}^-2, 0, 2, 4, 6, \ldots\}$

7060 Least common multiple
(See 1200, 1300)

The *least common multiple* (also lowest common denominator, LCD, of fractions) of two or more natural numbers is the least natural number exactly divisible by all of the numbers.

Ex. The LCM of 4, 10 and 12 is 60.

ELEMENTARY NUMBER THEORY

Unique factorization (prime factorization) 7070

Unique factorization or complete factorization occurs when the number is expressed as the product of its prime factors.

Ex. $3 \times 4 = 12$ shows 3 and 4 as factors of 12 but the expression $3 \times 2 \times 2 = 12$ shows complete factorization.

Rules for divisibility 7080

All even numbers can be divided exactly by 2.

All numbers represented by numerals ending in 0 or 5 are exactly divisible by 5.

All numbers represented by numerals ending in 0 are exactly divisible by 10.

If the sum of the numbers named by the digits in a base 10 numeral is exactly divisible by 3 then the number is divisible by 3.

Ex. 288 is divisible by 3 since the sum $2 + 8 + 8$ or 18 is divisible by 3.

<u>Proof</u>: $2 \times (100) + 8 \times (10) + 8 =$

$2 \times (99 + 1) + 8 \times (9 + 1) + 8 =$

$2 \times 99 + 2 + 8 \times 9 + 8 + 8 =$

$(2 \times 99) + (8 \times 9) + 2 + 8 + 8 = 288$

The first two terms are divisible by 3; then the number is divisible by 3 if $(2 + 8 + 8)$ is divisible by 3.

Rules for divisibility by 4,6,8 and 9 are often used, also.

NUMBER SEQUENCES AND PATTERNS

General number sequences and patterns 7082

Number Patterns and Relationships

NUMBER SEQUENCES AND PATTERNS

Number sequences are numbers given in some order, usually according to a pattern.

Ex. 1,1,2,3,3,4,5,5,6,··· Ex. 3,2,4,3,5,4,6,5,···

Ex. 0,3,8,15,24,··· Ex. 1,3,4,7,11,18,29,···

7090 Arithmetic progressions
(See 0075, 0080)

An arithmetic progression is a sequence of numbers each differing from the preceding number by a fixed amount.

Ex. 3,6,9,12···the constant difference is 3;

Ex. 8,6,4,2,···the constant difference is $^{-}2$.

Use code 7090 when arithmetic progressions are so called by the authors. For skip counting in primary grades use code 0080.

7100 Geometric progressions

A geometric progression is a sequence of numbers each of which differs from the preceding number by a constant factor.

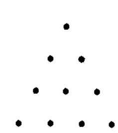
Ex. 1,3,9,27,···the constant factor is 3.

Ex. 32,16,8,4,2,1,½,¼,···the constant factor is ½.

7110 Triangular numbers

A triangular number is the cardinal number of a set of dots used in making triangular arrays beginning with one dot and continuing with rows of 2,3,4,··· dots.

Ex. Using the top two rows of the diagram, 3 is seen to be a triangular number. Using the top three rows 6 is seen to be such a number.



NUMBER SEQUENCES AND PATTERNS

Square numbers 7120

Square numbers are the cardinal numbers of square arrays.

Ex.

```
          • • •
  • •     • • •
  • •     • • •
```

4 and 9 are such square numbers.

Factorial numbers 7130

Factorial numbers are numbers symbolized by $n!$ or $\underline{|n}$ to indicate the product of a series of consecutive positive integers from 1 to the given number.

Ex. $3! = 1 \times 2 \times 3 = 6$

3! is read "factorial three." It may also be read as "three factorial."

Other special sequences 7150

Ex. Fibonacci numbers are numbers in the sequence 0,1,1,2,3,5,8,$\cdots$. Each number beginning with the third is obtained by finding the sum of the two preceding numbers. Leonardo Fibonacci was a mathematician of the 13th Century who wrote treatises on the theory of numbers. His name was attached to the above sequence.

Pythagorean triples 7155
(See 5320)

A *Pythagorean triple* is a triple of whole numbers which can be the lengths of the sides of a right triangle.

Ex. 3,4,5 since $3^2 + 4^2 = 5^2$

Ex. 5,12,13 since $5^2 + 12^2 = 13^2$

NUMBER SEQUENCES AND PATTERNS

7160 Other special patterns (including short cuts)

Code with an operation if possible.

Ex. To multiply by 25 quickly, multiply by 100 and divide by 4 (actually multiplying by $\frac{100}{4}$, another name for 25).

Ex. $45^2 = (40 + 5) \times (40 + 5)$
$= (40 \times 40) + (10 \times 40) + (5 \times 5)$
$= (50 \times 40) + 25 = 2025$

Using the short cut
$45^2 = (5 \times 4) \times 100 + 25$ or 2025

Code 7160 and 0700 (raising to powers and finding roots).

Ex. 15 × 15 = 225
35 × 35 = 1225
65 × 65 = 4225
75 × 75 = 5625
95 × 95 = 9025
45 × 45 = 2?25
85 × 85 = ????

There is an easy way to find the product when a number is multiplied by itself if the numeral for the number has a 5 in the ones place. Can you see the pattern?

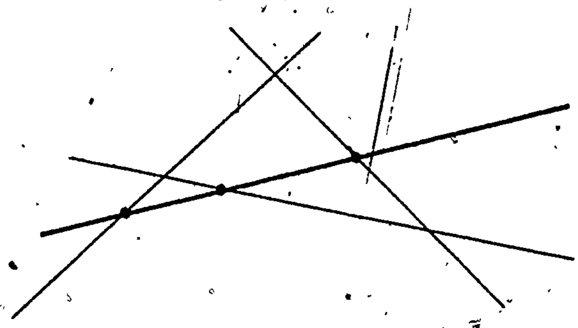
Let t = tens' digit
Let 5 = ones' digit

$(t + 1) \times t \times 100 + (5 \times 5) = N$

Ex. Since 3 + 7 = 10, then 13 + 7 = ?
Since 8 + 7 = 15, then 18 + 7 = ?
Since 8 + 9 = 17, then 18 + 9 = ?
Since 9 + 6 = 15, then 19 + 6 = ?
Since 4 + 9 = 13, then 14 + 9 = ?

NUMBER SEQUENCES AND PATTERNS

Ex.



In this figure there are ? lines. The heavy line intersects each of the other lines in ? point(s).

Does each line intersect every other line in the same number of points?

To find the greatest number of points of intersection determined by 4 lines, multiply the number of lines, ?, by the number of points of intersection on each line, ?, and then divide by 2: There are $\frac{? \times ?}{2}$, or $\frac{?}{-}$ points.

Try this with 5 lines, 6 lines, 3 lines, *n* lines.

TOPIC VII:

Other Topics

Ratio and proportion

8000 A. Ratio

(See 5255)

A ratio is a comparison of two numbers by division.

A ratio is a fractional number used to compare the cardinal numbers of two disjoint sets.

Ex. The ratio of set A to set B is $\frac{2}{5}$.

A = ●●

B = ○○○○○

The ratio is a comparison between two quantities which have the same dimensions, expressed in the same unit.

Ex. Larry has 4 books and John has 7 books. The ratio of Larry's books to John's is 4 to 7 or $\frac{4}{7}$.

A ratio of 1 to 2 can be represented by any member of the set $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots$. It may be expressed as 1 to 2, $\frac{1}{2}$, 1:2.

A statement which shows that two ratios are equal is called a *proportion*.

Ex. $\frac{2}{3} = \frac{4}{6}$ Ex. $\frac{2}{3} = \frac{x}{18}$

B. Direct and inverse variation 8001

Whenever the *quotient* of two variables is a constant, we say that they *vary directly*.

An example of *direct variation*: $\frac{d}{r} = 7$ or $d = 7r$

Whenever the *product* of two variables is a constant, we say that they *vary inversely*.

An example of *inverse variation*: $r \times m = 560$ or $r = \frac{560}{m}$ or

$m = \frac{560}{r}$

The constant is called the *constant of variation*.

C. Proportion (including rate pairs) 8002

A rate is a comparison between two quantities having different dimensions such as miles per hour.

Ex. If one candy bar costs 6¢, 2 candy bars cost ?¢.

Note: Most verbal problems using multiplication involve the concept of rate.

See 4035

Per cent

A. Meaning and vocabulary (See 3035) 8004

B. Developed through use of ratios 8005

C. Developed through use of equations 8006

D. Developed through use of the formula $p = b \times r$ (percentage equals base times rate) 8007

Other Topics

Ex. What is 8% of 62?

62	b (base)
.08	r (rate)
4.96	p (percentage)

8008 E. Computation related to per cent

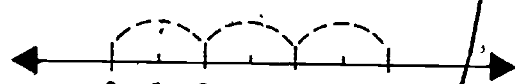
Graphs

(See 8050)

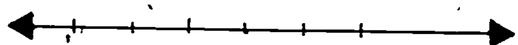
8020 A. Solution sets of equalities and inequalities on the number line

Ex. $3 \times \square = 6$ The solution set is {2}.

The number line shows



Graph the inequality $3 \times \square < 10$ if the universal set is the set of whole numbers. The solution set by dots is {0,1,2,3}. The number line graph is shown.

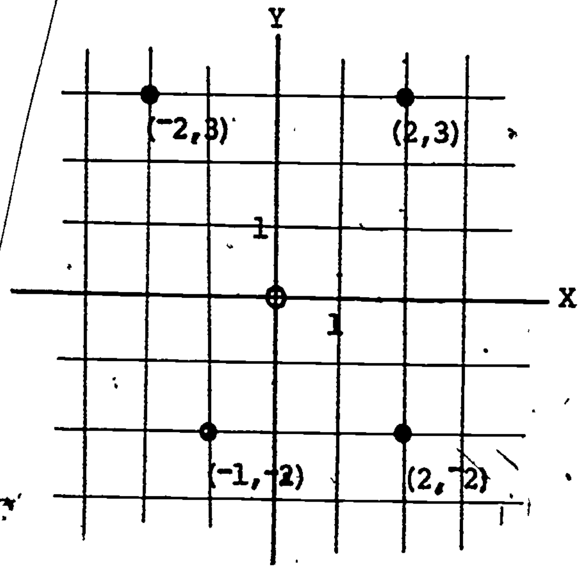


8030 B. Ordered pairs on a coordinate plane

Over 300 years ago Descartes envisioned a plane (surface) on which pairs of numbers were used to locate points. This plane is called the Cartesian or coordinate plane. Ordinary graph paper illustrates such a plane. The pairs of points are ordered so that the first number represents the horizontal or x distance and the second number represents the vertical or y distance.

Ex. The x and y axes drawn on the plane may be considered as two number lines with the 0 point at their intersection. The ordered pair (−2,3) locates a point 2 units to the left of the 0 point and 3 units above it.

Ex.

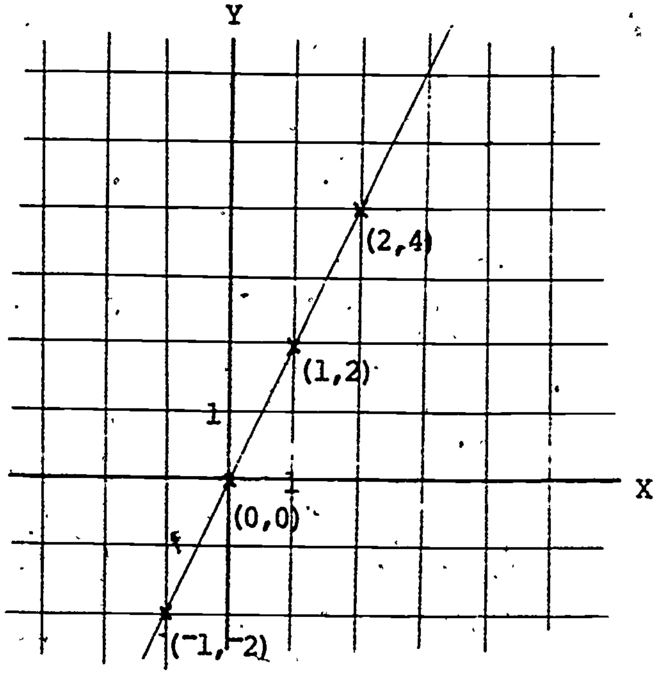


Use 8030 to code the mechanics of graphing ordered pairs.

C. Solution sets of equalities and inequalities on a coordinate plane 8040

Ex. *Equality* $\{(x,y)\,|\,y = 2x\}$

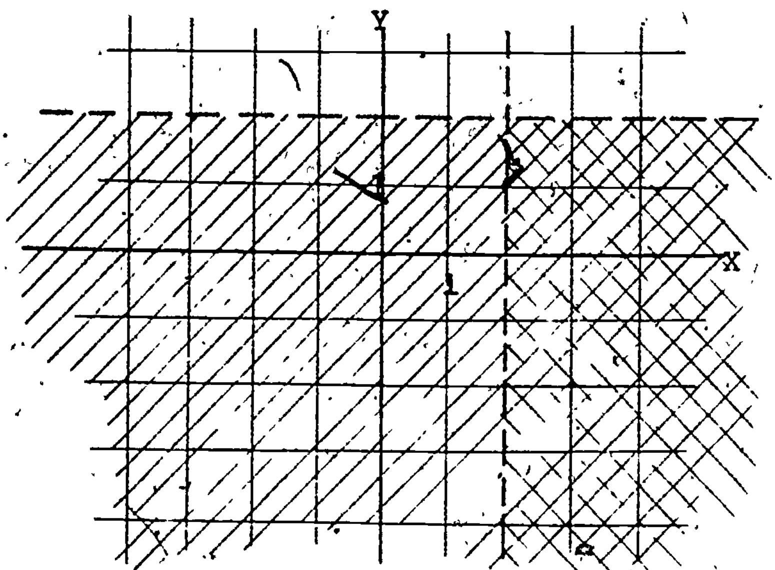
x	y
$^-1$	$^-2$
0	0
1	2
2	4
3	6
.	.
.	.
.	.



Find ordered pairs like those given. Plot them on the coordinate plane. Connect the points. A straight line will result.

The coordinates (numbers comprising the ordered pair) of any point on the line serve as a *solution set* for the given equation, or equality.

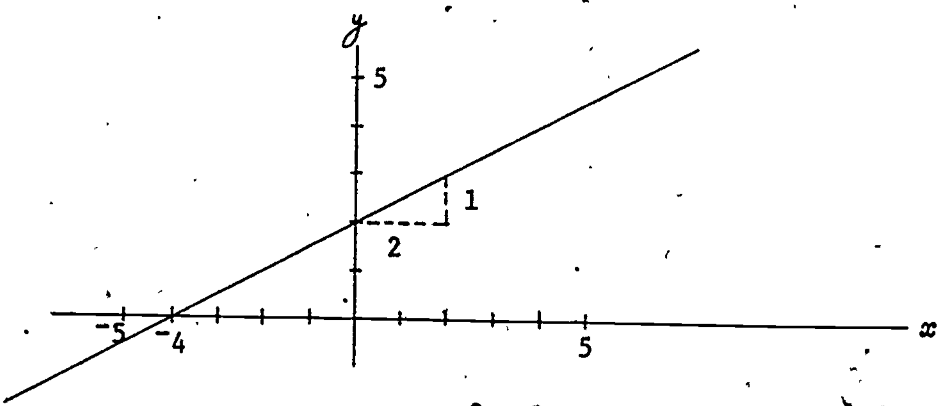
Ex. *Inequalities* $\{(x,y)\,|\,x > 2,\ y < 2$



Since $x = 2$ is not part of the graph, that line is dotted. For the same reason the line $y = 2$ is dotted. Where the two sets of arrows cross each other we have an area extending indefinitely to the right and downward. The coordinates of any point in this area serve as a solution set for the given inequality.

D. Slope, intercept, etc.

The slope of a line is the ratio of the *rise* to the *run*. 8042



The slope of the line is $\frac{2}{4} = \frac{1}{2}$

If the line intersects the y-axis at $(0,b)$, b is called the y-intercept.

In the example above, the y-intercept is 2.

If the line intersects the x-axis at $(a,0)$, a is called the x-intercept.

In the example above, the x-intercept is $^-4$.

In an equation written in the form $y = mx + b$, m is the slope and b is the y-intercept.

Ex. $y = \frac{2}{3}x + 9$

The slope is $\frac{2}{3}$

The y-intercept is 9.

Other Topics

Descriptive statistics

8050 A. Frequency tables, charts, graphs (bar, line, circle, dot, picture, etc.)

8060 B. Measures of central tendency: average, mean, mode, median

The arithmetic *mean* of a sequence of numbers is an average and is found by dividing the sum of the numbers by the number of items in the sequence.

Ex. The A.M. of 4,8,10 and 16 is (4 + 8 + 10 + 16) ÷ 4.

The *mode* of a sequence of numbers is the number or category that occurs most often.

The *median* of a sequence of numbers is the middle score in the sequence after the scores have been arranged from lowest to highest or highest to lowest. The median of the scores 5,6,8,12,18,20 and 24 is 12.

The median of scores 4,5,6,8,11,12,18,24 is assumed to be ½ the sum of the two middle terms 8 and 11. ½ (8 + 11) = 9½.

8070 C. Measures of variability: range, quartiles, percentiles, average deviation, standard deviation

The range of a sequence of numbers is the interval between the least and the greatest of a set of quantities.

Ex. The range of the series 1,3,7,10,15 is 15 - 1 or 14.

The first quartile Q_1 is the point below which lie 25% of the scores.

The third quartile Q_3 is the point below which lie 75% of the scores.

The 20th percentile is the point below which lie 20% of the scores.

The 50th percentile is the point below which lie 50% of the scores.

The average deviation is the arithmetic mean of the deviations of all the separate measures from the arithmetic mean. It is found by using the formula

$$A.D. = \frac{\Sigma|x|}{N}$$

The absolute value of the sum of the deviations divided by the number of deviations is the average or mean deviation.

The standard deviation for ungrouped data is found by using the formula

$$\sigma = \sqrt{\frac{\Sigma x^2}{N}}$$

where x is the deviation of each score from the mean, Σx^2 is the sum of the deviations squared, N is the number of terms.

Permutations and combinations

8075

Each different arrangement or ordered set of objects is a *permutation* of those objects.

Ex. List the permutations of 2, 3 and 4 using all three digits.

Answer: 234, 243, 324, 342, 423, 432

Ex. Imagine that there are three empty seats in a bus and two people get on. In how many ways can they pick seats?

Answer: 6

A *combination* is a set or collection of objects in no particular order.

Ex. How many committees of three members can be formed from three people?

Answer: 1

Ex. Three people, A, B and C, enter a room. Each one shakes hands with the others. How many handshakes?

Answer: 3

Other Topics

Probability

8080 A. Intuitive concepts

If several events are equally likely to happen, the chance (probability) that a given event will happen is the ratio of the favorable possibilities to the total possibilities.

The probability that a 3 will show on one toss of a die is $\frac{1}{6}$. Only one 3 can appear. Any of six numerals may appear.

8082 B. Formal concepts

Ex. A bag contains five marbles. On ten draws you got a green marble ten times. What do you estimate the chance of drawing a green marble? a red marble?

Ex. There are 30 marbles in a bag. If five are green, 12 are blue and three are red, what is the chance of drawing a blue one?

Other mathematical systems (finite or infinite)

8100 A. Modular arithmetic (clock arithmetic)

Modular arithmetic is based upon a set of finite numbers.

Ex. The usual clockface has only 12 numbers. Hence, in that number system we may write $8 + 5 = 1$. A movement of the hand 5 spaces beyond 8 brings the hand to 1.

Note: Use of the clock to teach the base 10 system of numeration is coded 3050. Telling time is coded 6050. Use this code for finite mathematical systems only.

8110 B. Without numbers

Letters or geometric figures may be used.

8120 C. Other

Logic

A. Reasoning 8130

Logic may be described roughly as the study of necessary inferences or compelling conclusions.

Ex. Answer *yes* or *no*. Mary has two dimes. Ann has two nickels. Mary and Ann have the same amount of money.

B. Logic in depth 8135
(See 4097)

Use this code for work stressing syllogisms, truth tables, Venn diagrams (when used in logic), ...

Ex. Draw a conclusion from the following:

(1) If Mary is at school, then Susie is at home.

(2) If Susie is at home, then the bird is singing.

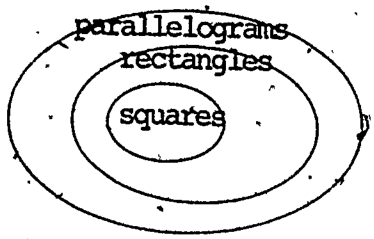
(3) Mary is at school.

Ex. Draw a Venn diagram for the following:

(1) All rectangles are parallelograms.

(2) All squares are rectangles.

(3) Therefore all squares are parallelograms.



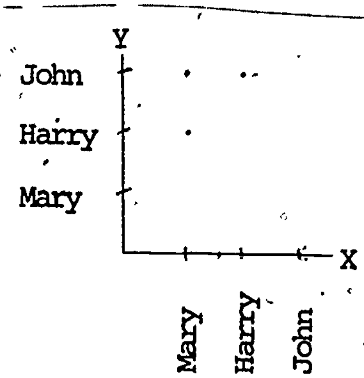
8140 Relations and functions

A relation is often defined as a set of ordered pairs. Thus {(Mary, John), (Harry, Sue), (1,2), (8,6), (a,b)} is a relation even though the elements appear to be selected at random. However, the selection of members of the set is usually made on the basis of some meaningful relationship.

Ex. Suppose it is known that John is 4 years old, Harry is 6 and Mary is 7, and the relationship "is older than" is given.

Thus the relation is {(Mary, Harry), (Mary, John), (Harry, John)}.

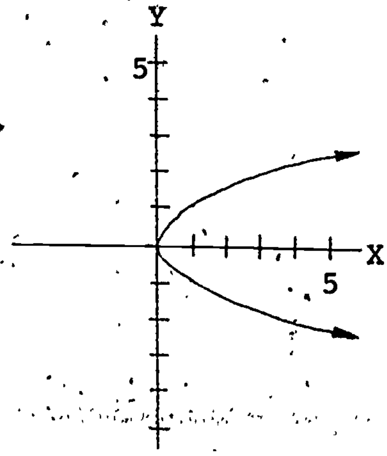
The graph of this relation:



Ex. The pairs of real numbers that make $y^2 = x$ a true statement form a relation that can be written as $\{(x,y) \mid y^2 = x\}$.

Some of the members of this set are (0,0), (1,1), $(1,^{-}1)$, (4,2).

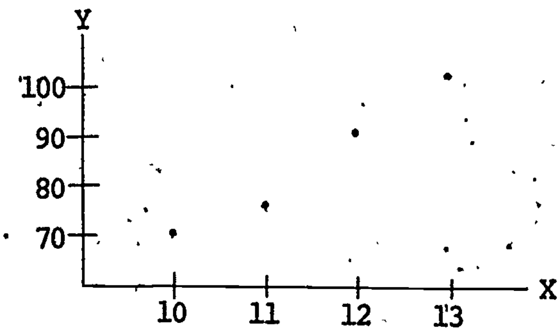
The graph of this relation:



Ex. A set of ordered pairs may be formed with the first number being John's age and the second his weight in pounds on his birthday:

{(10,70), (11,75), (12,89), (13,100)}

The graph of this relation:



Example 3 differs significantly from examples 1 and 2. In example 3, *only one point* is graphed above (or below) one location on the x-axis, whereas in the other two examples, more than one point appears above (or below) the same point on the x-axis. Relations such as that given in example 3 are single-valued relations and are called *functions*.

Other Topics

A *function* is a relation such that for each first value there is one and only one second value.

Ex. $\{(1,2), (8,3), (7,3)\}$

Ex. $\{(1,2), (8,5), (7,5), (4,3)\}$

Ex. $\{(r,c) \mid c = 2\pi r\}$, where $r > 0$

This code is to be used for definitions of, and for graphing for the purpose of illustrating the meaning of, the terms *relation* and *function*.

8145 Domain and range

The *domain* of a relation (or function) is the set of first members of each pair.

The *range* of a relation (or function) is the set of second members of each pair.

Ex. Given the relation $\{(6,5), (8,7), (2,6)\}$,

The domain is: $\{2,6,8\}$

The range is: $\{5,6,7\}$

8150 Estimation
(See 3100)

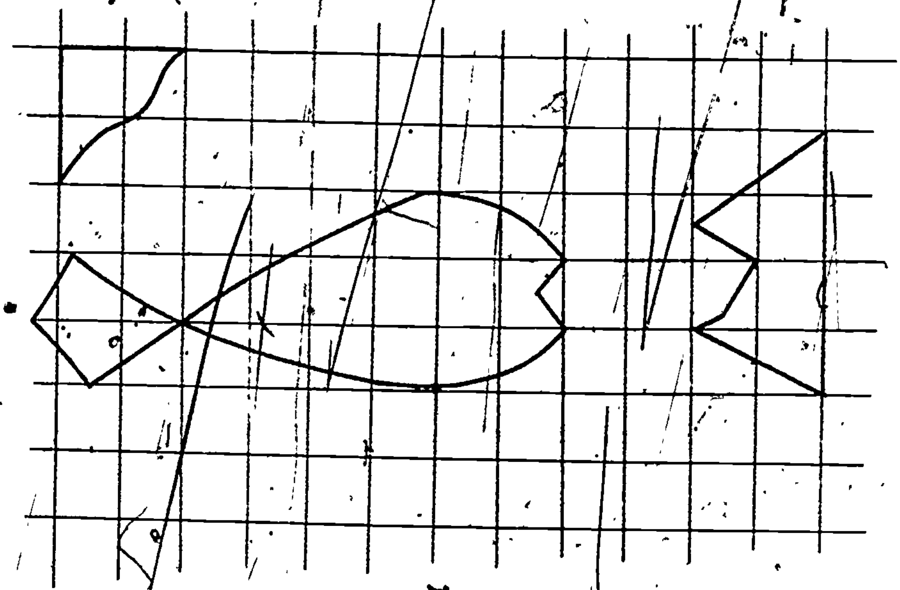
An approximate (estimated) answer to a problem can often be found by using rounded numbers and mental computation.

Ex. The sum of 428, 365 and 215 is approximately

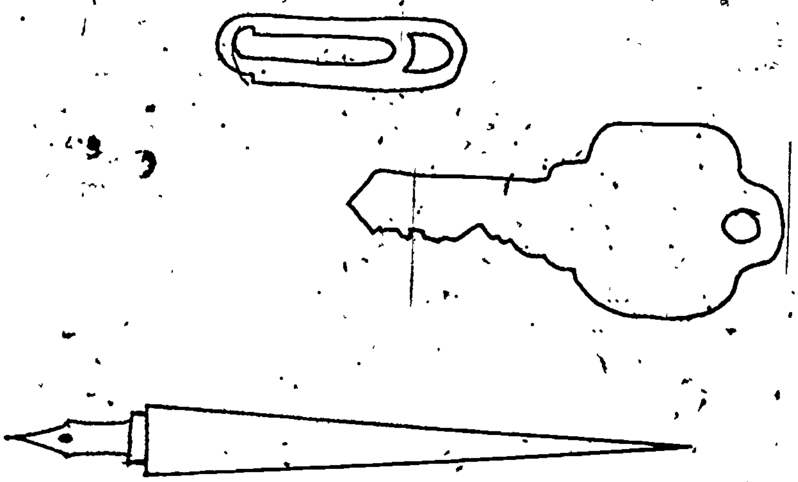
$400 + 400 + 200 = 1000$.

$$\begin{array}{r} 428 \\ 365 \\ +\ 215 \\ \hline 1008 \end{array} \leftarrow \text{actual sum}$$

Ex. Estimate the area of each region.



Ex. Estimate the length of each object to the nearest half inch.



Other Topics

8160 **Properties of relations**
(See 8140)

Suppose a set of elements and some rule for pairing them are given and a set of ordered pairs (a relation) is formed. If the given rule permits every element to be paired with itself, the relation is said to be *reflexive*.

Ex. Given the set of whole numbers and the rule *is equal to*. Since every number is equal to itself, the relation formed is reflexive.

If a and b are any two elements and if a can be paired with b then b can also be paired with a, the relation is said to be *symmetric*.

Ex. Given the rule *is the sister of* and two girls, Alice and Mary, who have the same parents. The relation is {(Alice, Mary), (Mary, Alice)}. Since Alice is the sister of Mary, and Mary is also the sister of Alice, the relation is symmetric.

If a, b and c are any 3 elements of a set and if a can be paired with b and b can be paired with c, then a can be paired with c, the relation is said to be *transitive*.

Ex. Given the children in a classroom and the rule *is taller than*. The relation so defined is transitive since if Mary is taller than Harry and Harry is taller than John, then Mary is taller than John.

If numbers are paired with the rule *is equal to*, the relation formed is reflexive, symmetric and transitive.

Ex. If people are paired according to the rule *is taller than*, the relation formed is transitive but it is not reflexive (since a person cannot be taller than himself) and it is not symmetric (because if Ann is taller than Betty, Betty cannot be taller than Ann).

8170 **Mathematical sentences**
(See 4125)

An arrangement of symbols indicating that a relationship exists between two or more things. The sentence contains at least two symbols for numbers, points, sets or the like and a relation symbol. The most common relation symbols are =, > and <.

Ex.

	symbol for thing	relation symbol	symbol for thing
Equations:	$2 + x$	$=$	3
	$3y$	$=$	18
Inequalities:	2	$>$	1
	$\frac{x}{y}$	$<$	7
	$3 + 5$	$\neq$	10
Other:	$\overleftrightarrow{AB}$	$\perp$	$\overleftrightarrow{CD}$
	$\{1,2\}$	$\subset$	$\{1,2,3,4\}$

Kinds of Sentences:	Open Sentence	Statement
	$x + y = 13$	$2 + 11 = 13$
	[illegible] $9 = 36$	$4 \times 9 = 36$
	6×5 [illegible] 24	$6 \times 5 > 24$

Developmental work with problem-solving may be classified under code 8170. Problem-solving (application) should be coded under the operation involved.

Application of mathematics to other subjects 8180

In textbook analysis, use only with at least one other code.

Flow charts 8190

If a flow chart shows instructions for a non-mathematical process, code 8190.

Other Topics

If a flow chart shows instructions for a mathematical process, code 8190 and at least one other code.

8200 History of mathematics

Use this code for biographies of mathematicians and other historical materials. For historical mathematics, code the mathematical concept.

Ex. "The Greek mathematician Eratosthenes, who lived over 2000 years ago, invented a way of sifting out the prime numbers from the other natural numbers." Code 8200

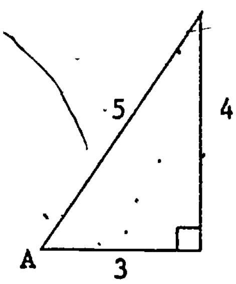
Code actual work with the sieve of Eratosthenes 7010.

8220 Trigonometry

A. Definition of trigonometric ratios

Use this code for the definitions of trigonometric ratios and for direct applications of the definitions.

Ex.



Give the value of sin A. Answer: $\frac{4}{5}$

8230 B. Numerical trigonometry

Ex. A right triangle has an acute angle with a measurement of 77°. The length of the leg opposite the 77-degree angle is 36 cm. Find the length of the hypotenuse to the nearest centimeter.

ALGEBRA

Basic concepts

A. Zero, the identity element for addition 8502
(See 0170, 1180, 2090)

B. One, the identity element for multiplication 8503
(See 0440, 1400, 2200)

C. The distributive property 8505
(See 0430, 0575, 1390)

Equations or inequalities

A. Linear in one variable 8510

Ex. Find each solution set:

$n + 4 = 6$

$6k = 24$

$9y > 36$

$4(x-5) = x + 7$

$3y - 175 < 200$

For graphs, see 8020

B. Linear in two or more variables 8515
(See 8040)

Ex. The sum of two numbers x and y is 13. The difference between x and y is 5. What are the numbers?

C. Quadratic in one variable 8520
(See 8040)

Ex. Name the two solutations of $x^2 + 4x = 0$.

Ex. Solve $(x - 4)(x - 1) = 0$ and $x \neq 4$.

8590 Order of operations

Ex. Which is larger, a or b?

$a = 5 + (4 \times 2)$

$b = (5 + 4) \times 2$

Answer: $a = 5 + 8 = 13$

$b = 9 \times 2 = 18$

$b > a$

Ex. Find the value of $5 + 4 \times 2$.

Answer: 13

When no symbols are used, the rule is to do operations in the following order:

first, powers and roots

second, multiplication and division

third, addition and subtraction.

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TOPIC VIII:

Summaries

General review 9000

> Review codes are to be used only if the author states (in teacher text or pupil text) that this material is review. In most cases, the word *review* will be used, but there may be an equivalent expression such as *maintenance of skills*.
>
> Use code 9000 if more than two mathematical content topics are presented in a single review lesson and one or two items of content cannot be identified as of major importance.
>
> Sets of supplementary pages at the end of a book are coded 9000 for general review or 9020 through 9111 for specific review unless these pages are intended for practice prior to mastery.

Test 9010

REVIEWS

Properties of and basic operations with whole numbers 9020

Properties of and basic operations with fractional numbers 9030

Properties of and basic operations with integers 9040

Numeration 9050

Sets 9060

Geometry 9070

Summaries

REVIEWS

9080 Measurement

9090 Number patterns and relationships

9101 Ratio and proportion

9102 Per cent

9103 Graphs

9104 Statistics

9105 Probability

9106 Finite mathematical systems

9107 Logic

9108 Relations and functions

9109 Estimation

9110 Properties of relations

9111 Mathematical sentences (equations)

Use when specifically being reviewed.

Mathematical sentences (equations) will be used with many topics and especially with problem solving. The lessons are then coded according to the topic being studied.

	Summaries
Algebra	9112
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Bibliography - student	9810
Bibliography - teacher	9820
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Glossary	9840
Index - student text	9850
Index - teacher text	9860
Introduction - foreward preface notes to teacher philosophy and series development information about authors	9870
Mathematics text for teacher inservice material	9880
Overview of Program - Grade Level (survey)	9890
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Vocabulary (no direct mathematical knowledge needed)	9981

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